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A Real-Time Algorithm for Assessing Inhomogeneities in Fabrics

Cloudiness constitutes an important quality parameter of nonwoven fabrics. We show that it can be graded using a specific Laplacian pyramid decomposition which has to be modified at the boundaries. An algorithm is presented which calculates a quality measure as a weighted mean of the variances at all pyramid levels, where the weights are adapted to the human perception of cloudiness. The obtained results reach the quality of a human assessor. The method is easy to implement and it can be used for online grading of the complete fabric production of a plant.

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Introduction

A large variety of industrial products, ranging from carpet to baby napkins, make use of nonwoven, fleece-like fabrics. For all these products, the fabric quality plays an important role. It is deteriorated by inhomogeneities such as clouds and stripes.

Clouds result from isotropic fibre agglomerations, thus having no preferred directions. Since large clouds do not look very pleasing, the scale of the cloudiness gives a measure of the optical appearance of the fabric. Moreover, the areas with the lowest density charaterize the mechanically weakest parts of the fabric.

Stripes consist of adjacent fibres preferring a certain direction. Therefore, the orientation of the stripes characterizes the fabric anisotropy, which has a strong impact on its mechanical properties.

The fabric producing company which brought this problem to our attention takes online images of their fabrics with a resolution well-suited for estimating the cloudiness. For evaluating the stripes, more advanced methods have to be applied which require a higher resolution. This cannot be investigated during the production process anymore. In the present paper, we shall therefore focus on the cloudiness.

In order to obtain objective, reliable quality measures for cloudiness, a suitable mathematical model is needed. On one hand, such a model would be useful for the internal assessment during the production, e.g. for surveying the product quality, for determining machine cleaning cycles and for comparing different machines. On the other hand, objective quality criteria provide standards for the customers and help them to specify their demands in a proper way.

A good model should be exact enough to match the human perception of cloudiness and it should allow fast algorithms which enable the company to perform grading during the production. In the present paper we shall discuss such a model and present a suitable real-time algorithm.

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It should be noted that our problem is quite different from questions of defect detection in textile fabrics which are treated for instance in Cohen *et al*. [1] and Neubauer [2]. For textile fabrics one is interested in finding well-located irregularities in woven structures. For nonwoven fabrics one is concerned with inhomogeneities which happen as stochastic fluctuations at the entire area of a fleece. Nevertheless, also in the latter case we can base our studies on a number of previous approaches in this field.

First, it should be observed that simple ideas like calculating the variance or the entropy of an image are not sensitive enough for our purpose, since they are independent of the ordering of the grey values. Hence, they cannot distinguish between multiple small clouds and one large cloud. In both cases, they yield the same amount of inhomogeneity. Since for the human observer there is a big difference between these two cases, it becomes clear that a suitable model must take into account the *scale* of a cloud.

Early suggestions in this direction were made by Neunzert and Wetton [3], who proposed using the discrepancy as a cloudiness criterion. Roughly speaking, the discrepancy measures the largest cloud or the largest hole in the fabric. A fast algorithm for two-dimensional (2D) interval discrepancy was presented by Hackh [4]. He reported that the results were still too coarse and not always selective enough for the desired purpose.

Stark [5] took into account the scale character of the cloudiness by analysing the fabric in a wavelet basis. Since his method did not exploit the main feature of wavelets, the localization in frequency *and* space, it seems that simpler techniques such as Fourier analysis suffice as well. Moreover, the grading step of this model was based on a fractal dimension assumption: it was claimed that the variance depends in a linear way on the scale. Recent experiences do not confirm this hypothesis.

Weickert [6] suggested processing the fabric image using nonlinear anisotropic diffusion in order to visualize clouds and the main stripes simultaneously. Although the results were fairly promising, the proposed method is not yet fast enough for online assessment. Furthermore, there seems to be no need to process both quality relevant features simultaneously. In Weickert [7] a preprocessing strategy based on nonlinear diffusion is discussed which is especially designed for the enhancement of coherent structures such as stripes.

In the present paper it is shown that linear diffusion suffices for evaluating the cloudiness. The linear diffusion process is used implicitly in the image description by means of a modified Laplacian pyramid. As a quality criterion of the cloudiness, it is proposed to use a weighted mean over the variances of all scales, with weights adapted to the human impression of cloudiness.

The paper is organized as follows. In the following section we shall briefly review the concept of image pyramids. We will see that the commonly used Laplacian pyramid has to be modified at the boundaries in order to be suitable for our task. Moreover, we shall point out the differences to other multiscale representations such as Fourier or wavelet analysis. Later sections describe the proposed quality criterion for the cloudiness of a fabric, discuss some experiments, and on algorithmic details. The paper concludes with a summary. Some preliminary results of our research have been presented at a mathematical conference [8].

Multiscale Analysis with the Laplacian Pyramid

Gaussian and Laplacian pyramid

For simplicity, we first restrict ourselves to the one-dimensional (1D) case in order to sketch the pyramid concept in image processing. Due to its separability, the 2D case follows immediately from the 1D case. For more details on pyramids, see for instance Burt and Adelson [9], Contoni and Ferretti [10], Jolion and Rosenfeld [11], and Rosenfeld [12].

We start by defining a (grey value) image as a vector $u = (u_0, \ldots, u_{2^N})^T$. We say that a vector is of level k if it consists of $2^k + 1$ components for $k \geq 1$, and of 1 component for $k = 0$. We consider linear interpolation operators A^k from level k to level $k + 1$, which are given by the matrices

$$A^k := \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & \cdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & \cdots & 0 & 0 & 1 \end{pmatrix} \quad (k \geq 1),$$

$$A^0 := \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Note that their row sums are always 1, and that the column sums never vanish. A^k is a $(2^{k+1} + 1) \times (2^k + 1)$ matrix for $k \geq 1$.

Next we define restriction operators R^k from level k to $k - 1$ via

$$R^k := \begin{pmatrix} \frac{2}{3} & \frac{1}{3} & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & \cdots & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & \cdots & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \frac{1}{3} & \frac{2}{3} \end{pmatrix} \quad (k \geq 1),$$

$$R^1 := (\tfrac{1}{3}\ \tfrac{1}{3}\ \tfrac{1}{3}).$$

R^k is a $(2^{k-1} + 1) \times (2^k + 1)$ matrix. Apart from R^1 and boundary points, the restriction operators consist of a convolution with the binomial mask $[\frac{1}{4}, \frac{1}{2}, \frac{1}{4}]$ followed by a coarser subsampling. In multigrid analysis, they are called full weighting operators [13]. The preceding convolution mask can also be regarded as an explicit finite difference scheme to the diffusion equation $u_t = u_{xx}$ with mesh ratio $\Delta t/(\Delta x)^2 = \frac{1}{4}$; see for instance [14]. For this ratio, the explicit scheme inherits several important smoothing properties of the continuous diffusion equation such as nonnegativity, a maximum–minimum principle, preservation of nontonicity, and it diminishes the total variation.

Using the preceding family of restriction operators, we obtain a Gaussian pyramid $\{v^N, \ldots, v^0)$ of u by

$$v^N := u, \tag{1}$$

$$v^{k-1} := R^k v^k \qquad (k = N, \ldots, 1). \tag{2}$$

The Gaussian pyramid gives a sequence of low-pass filtered versions of u, whose size is reduced in each step by a factor which is close to $1/2$. If we interpolate every successor in the Gaussian pyramid and subtract it from its predecessor, we get a band-pass representation of u:

$$w^k := v^{k-1} A^{k-1} v^{k-1} \qquad (k = N, \ldots, 1), \tag{3}$$

$$w^0 := v^0. \tag{4}$$

$\{w^N, \ldots, w^0\}$ is the Laplacian pyramid of u. Its highest frequency component is w^N.

Modification at the Boundaries

Most literature on image pyramids does not address the question of how to choose the restriction operator at the boundaries. Other choices than the one that we have already seen may cause undesirable effects near the boundaries of the Laplacian pyramid (such as strong oscillations), and the pyramid levels $w^N, \ldots, w^1$ may not have zero mean. The latter phenomenon is rather untypical for other band-pass representations such as Fourier or wavelet-based ones. It can only happen if the restriction with the subsequent interpolation does alter the average grey value of the image. This is also undesirable if we think of the diffusion interpretation of this process: diffusion is based on the continuity equation and therefore, it is conservative. This indicates that the restriction operator has to be chosen very carefully at the boundaries in order to fit the interpolation operator well.

The following criterion shows how the restriction operator R^k should be related to the interpolation operator A^{k-1} to ensure that $w^k = v^k - A^{k-1} R^k v^k$ has zero mean. A proof for this criterion can be found in the appendix.

Selection criterion for restriction operators:

Let $A = (a_{ij})$ satisfy $\sum_j a_{ij} = 1$ and $\sum_i a_{ij} \neq 0$. Then, choosing $R = (r_{ij})$ such that

$$r_{ij} = \frac{a_{ji}}{\sum_l a_{li}} \tag{5}$$

guarantees that AR preserves the average grey value:

$$\sum_i y_i = \sum_i x_i \quad \text{for} \quad y = ARx. \tag{6}$$

One can easily check that the suggested restriction operators satisfy this criterion.

It should be noted that—in our case—a correct treatment at the boundaries is of crucial importance. As we will see in the next section, we need the variance of each pyramid level in order to grade the cloudiness. Especially at coarse levels (corresponding to large clouds), boundary pixels cause an important contribution to this value. Incorrect boundary treatment would be the source of significant deviations and misinterpretations.

Comparison with other multiscale methods

Like Fourier and wavelet transformation, the Laplacian pyramid gives a complete image representation which allows retrieval of the image entirely, see, for example, Burt [9]. Since this representation is completely performed in the spatial domain, we obtain the band-pass filtered versions immediately, no preceding reconstruction is necessary. In contrast to the Fast Fourier Transform (FFT), however, the computational effort is linear in the number of pixels. On the other hand, a Laplacian pyramid contains always some redundancy, since the pyramid representation is about $1/3$ larger than the original image. Although the frequency sepa-

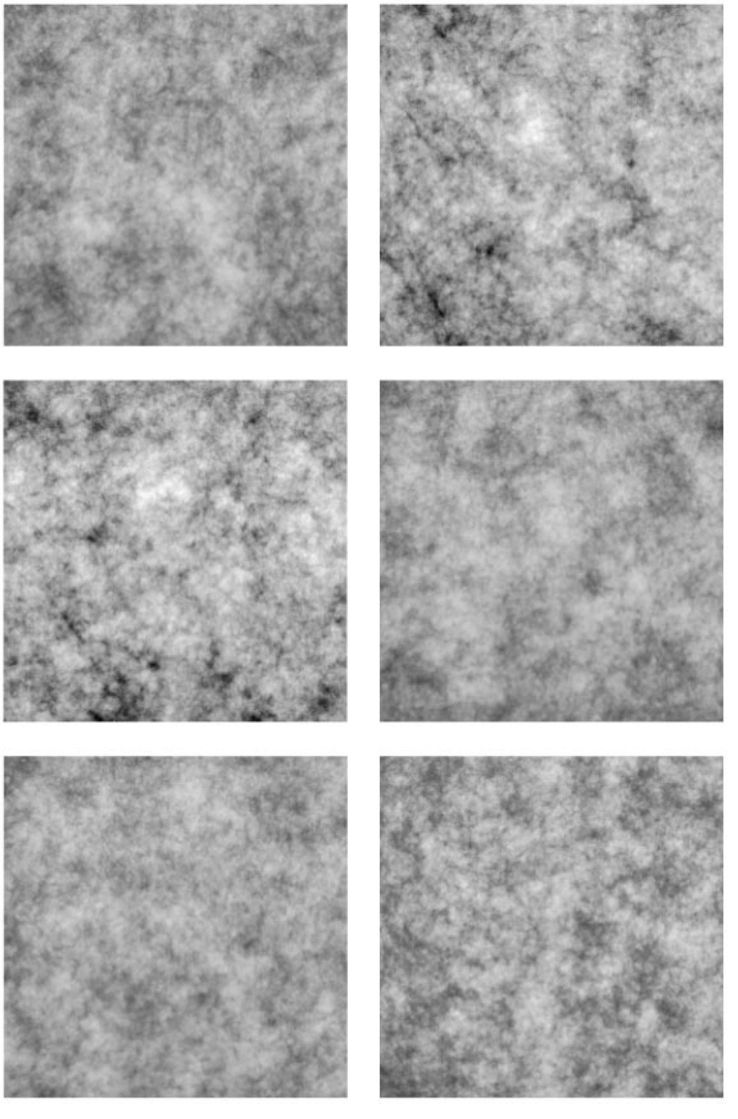

Figure 1. The test set.

ration between neighbouring scales is not as strict as in the wavelet and Fourier case, we shall see that it is sufficient for our purposes. Our experiments have indicated that the Laplacian pyramid is sufficiently robust with respect to translations (a frequent problem when using wavelets) and it performs better at boundaries than wavelet and Fourier methods. A correct boundary treatment would be one of the main problems when using wavelet analysis. For the FFT, discontinuities at the periodic extensions of images may lead to anisotropic artifacts, which are not apparent in the original image.

Recapitulating, besides its simplicity and speed, the robust behavior near boundaries is the main reason for preferring Laplacian pyramids to other multiscale approaches for the present problem.

The Quality Criterion for Cloudiness

Having a band-pass representation of the fabric image by means of the Laplacian pyramid, one may take the variance σ_k^2 at some scale k as a measure of the cloudiness at this scale. Since our Laplacian pyramid levels are designed to have zero mean (except for $k = 0$), the variances can be obtained from

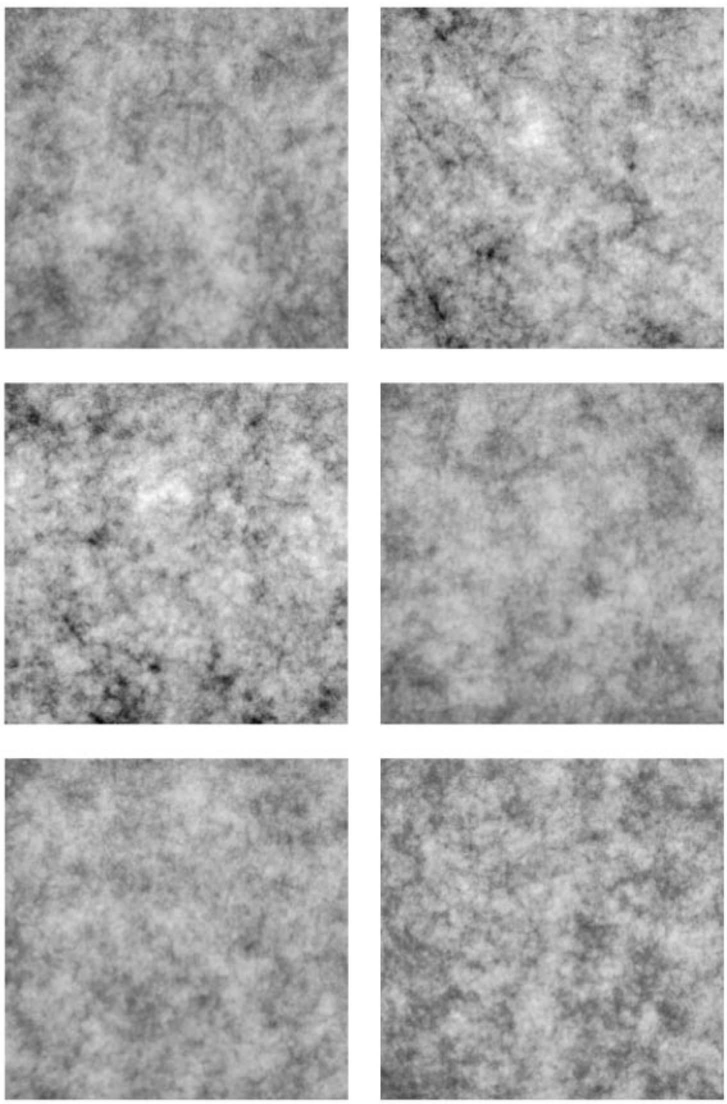

Figure 2. The test set ordered according to the quality criterion. From top left to bottom right: increasing cloudiness.

Table 1. Results for the test set consisting of six fabric images. σ_k^2 gives the variance at the k-th pyramid level, c denotes the resulting quality index, and r is the average ranking by 32 human assessors

Fabric	#5	#1	#4	#6	#2	#3
σ_8^2	20.99	21.26	20.39	36.69	49.60	54.16
σ_7^2	32.44	32.69	30.66	56.57	66.77	77.99
σ_6^2	42.30	44.68	40.48	75.81	80.96	104.92
σ_5^2	48.93	49.59	50.01	87.83	94.23	113.30
σ_4^2	52.86	52.84	57.97	75.42	94.87	103.86
σ_3^2	47.68	59.08	48.92	36.29	66.70	86.35
σ_2^2	23.60	27.96	44.83	31.55	57.16	75.15
σ_1^2	33.20	77.82	99.62	91.73	156.94	179.05
c	48.32	51.79	52.26	66.59	85.26	100.63
r	1.50 $\pm$0.56	2.28 $\pm$0.51	2.28 $\pm$0.45	4.38 $\pm$0.54	5.03 $\pm$0.17	5.53 $\pm$0.50

$$\sigma_k^2(u) = \frac{1}{2^k+1} \sum_{i=0}^{2^k} (w_i^k(u))^2 \qquad (k = 1, \ldots, N). \tag{7}$$

Now the question arises of how to assemble the variances σ_k^2, $k = 1, \ldots, N$ to a single value describing the human impression of cloudiness. One of the most natural ideas is to calculate a weighted mean. But how shall the weights be chosen?

To solve this question, an experiment was performed: 18 members of our department (with different ages, nationalities, and degrees of expertise) and 14 employees from the fabric-producing company were asked to classify six fabric images according to their visual appearance of cloudiness. The test set is depicted in Figure 1. The human assessors were supposed to grade the most homogeneous fleece with 1 point, the second most with 2 points and so forth. The bottom row of Table 1 shows the mean and the standard deviation of these rankings.

The result was fairly surprising: most of the candidates gave a very similar ranking. This indicates that there seems to exist a typical human impression of cloudiness independent of age, gender, cultural background and knowledge. It takes into account mainly inhomogeneities at middle scales. Smaller and larger clouds had significantly less influence on the assessment. For this reason, we may choose weights proportional to a Gaussian distribution which is centered around the middle scales and decreases rapidly towards small and large scales:

$$w(k) = \frac{\exp\left(-\frac{(k-\mu)^2}{2\sigma^2}\right)}{\sum_{j=1}^{N} \exp\left(-\frac{(j-\mu)^2}{2\sigma^2}\right)}. \tag{8}$$

The mean μ and the standard deviation σ are determined in order to fit the experimental data best. Thus, the final expression of the cloudiness c of a fabric u is

$$c(u) = \sum_{k=1}^{N} w(k) \sigma_k^2(u). \tag{9}$$

In the case of the test data set, the resulting values for σ_k^2 and c can be found in Table 1. We observe that one can get the same ranking as for a human observer. The result was not very sensitive with respect to the choice of the parameters μ and σ. In Table 1, the values $\mu := 4$ and $\sigma := 1$ were used. Figure 2 shows the test images ordered by increasing cloudiness.

The validity of this quality criterion was checked by comparing its results with human assessment of other data sets. In all cases, the ranking of the model was within the standard deviation of rankings by human experts.

Algorithmic Details

Parameters

A programme which is supposed to be used in industry should have only a small number of parameters, and they should have an intuitive meaning. Our programme requires four parameters: two of them are the pixel number M and the pixel size Δx, which follow from the resolution and the positioning of the camera. The other two determine an interval $[a, b]$ which gives the typical diameter range of clouds which are regarded as quality-relevant.

The parameters a and b are used to determine the mean μ and the standard deviation σ in the Gaussian weight function [Eqn (8)]. If $k(s)$ denotes the pyramid level k, which has highest sensitivity for clouds of scale s, then μ and σ are chosen such that $k(a) = \mu - \sigma$ and $k(b) = \mu + \sigma$. This gives

$$\mu := \frac{k(a) + k(b)}{2}, \tag{10}$$

$$\sigma := \frac{k(a) + k(b)}{2}. \qquad (11)$$

Now the question arises as to how to obtain the function $k(s)$. From an analysis in the Fourier domain one would expect that reducing the structure size s to $s/2$ would lead to an increment of k by approximately 1. However, since pyramids are not translation-invariant and clouds are rather a local stochastic phenomenon than a global, cosine-shaped function, we also made some stochastic simulations.

We analysed the behaviour of our Laplacian pyramid decomposition by applying it to synthetic images with cloud-like blobs of different sizes, which were placed at random locations within the image.

To express the structure size $s(k)$ where a pyramid level k attains its maximal sensitivity, we used the ansatz

$$s(k) = \alpha M \Delta x \cdot \exp(-\beta k). \qquad (12)$$

Fitting the parameters α and β to our experimental data yielded $\alpha := 0.914$ and $\beta := 0.6032$. Solving [Eqn (12)] for k gives

$$k(s) := \frac{1}{\beta} \ln\left(\frac{\alpha M \Delta x}{s}\right). \qquad (13)$$

Let us illustrate these considerations by a practical example with typical parameters: if a 512×512 image describes an area of 100×100 cm^2, then $M = 512$ and $\Delta x = 0.195$ cm. Taking into account clouds between $a = 5$ cm and $b = 15$ cm diameter leads to $k(a) = 4.817$ and $k(b) = 2.996$. This gives $\mu = 3.906$ and $\sigma = 0.910$. The fact that this is in good agreement with the fit parameters $\mu = 4$ and $\sigma = 1$ from the previous section gives evidence that the 32 assessors were indeed focusing on clouds between 5 and 15 cm.

Pyramid decompositions use most of their computation time for operations at high pyramid levels. Since the cloudiness is dominant at middle levels, one can gain a significant speed-up by downsampling the original image. Frequently we downsampled 512×512 images such that the highest pyramid level was 7.

Storage effort and execution times

The whole algorithm was implemented as a portable C programme using only ANSI features. Table 2 shows the measured storage effort and the execution times on an HP 9000/889 workstation.

Table 2. Storage effort and execution time for grading one fabric image on an HP 9000/889

Size M	Max. level N	CPU time (s)	Memory (MB)
128	6	0.007	0.2
128	7	0.018	0.8
256	6	0.012	0.9
256	7	0.023	1.4
256	8	0.075	2.7
512	6	0.093	3.2
512	7	0.112	3.4
512	8	0.185	4.2
512	9	0.446	11.4

Let us now discuss the practical consequences of this table for typical data sets. We observe that downsampling 512×512 images to a highest pyramid level of 7 and executing the grading algorithm allows to treat about 9 images per second. If one image depicts a fabric area of 1 m^2, it is possible to assess 9 m^2/s. This shows that many plants can grade their entire fleece production with such an online algorithm running on one workstation. In many cases, one or two fast PCs will suffice as well, and a significant speed-up can be achieved by using image sizes of 256×256 instead of 512×512. This has basically no influence on the grading results.

In the meantime, our industrial co-operation partner has been using this algorithm for many months. By comparing it with previously used offline methods and the results of human assessors, it has been concluded that this algorithm combines high reliability with good selectivity and online qualities.

Summary

The cloudiness of nonwovens is a scale-phenomenon which can be analysed using a Laplacian pyramid decomposition. The pyramid should be modified at boundaries in order to reduce errors induced by these values. As a measure of cloudiness, one may use a weighted average of the variances at all scales, with weights according to the human perception. The obtained model reaches the qualities of a human assessor, and it is fast enough for online grading. Implemen-

tations at our industrial cooperation partner have demonstrated its success as a real-time grading tool.

It appears that the scope of the presented method is not restricted to grading of the cloudiness of nonwovens. Our algorithms might also be applicable to similar assessment problems, for instance to online grading tasks within the paper and marble industry.

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Appendix

Proof of the selection criterion

Let $C := (c_{ij}) := AR$.

First we show that all row sums of C are 1:

$$\sum_j c_{ij} = \sum_j \sum_k a_{ik} r_{kj} = \sum_j \sum_k a_{ik} \frac{a_{jk}}{\sum_l a_{lk}}$$

$$= \sum_k a_{ik} \frac{\sum_j a_{jk}}{\sum_l a_{lk}} = \sum_k a_{ik} = 1.$$

Furthermore, because of

$$c_{ij} = \sum_k a_{ik} r_{kj} = \sum_k \frac{a_{ik} a_{jk}}{\sum_l a_{lk}} = \sum_k a_{jk} r_{ki} = c_{ji}$$

we know that C is symmetric. Thus, all column sums of C are 1 as well, and we have

$$\sum_i y_i = \sum_i \sum_j c_{ij} x_j = \sum_j \left(\sum_i c_{ij} \right) x_j = \sum_j x_j.$$

This concludes the proof.