Robust Estimation of Roughness Parameter in SAR Amplitude Images

Héctor Allende and Luis Pizarro

Departamento de Informática, Universidad Técnica Federico Santa María,
Casilla 110-V, Valparaíso, Chile
{hallende, lpizarro}@inf.utfsm.cl

Abstract. The precise knowledge of the statistical properties of synthetic aperture radar (SAR) data plays a central role in image processing and understanding. These properties can be used for discriminating types of land uses and to develop specialized filters for speckle noise reduction, among other applications. In this work we assume the distribution $\mathcal{G}_A^0$ as the universal model for multilook amplitude SAR images under the multiplicative model. We study some important properties of this distribution and some classical estimators for its parameters, such as Maximum Likelihood (ML) estimators, but they can be highly influenced by small percentages of 'outliers', i.e., observations that do not fully obey the basic assumptions. Hence, it is important to find Robust Estimators. One of the best known classes of robust techniques is that of M estimators, which are an extension of the ML estimation method. We compare those estimation procedures by means of a Monte Carlo experiment.

Keywords: Robust Estimation, SAR Images, Speckle Noise, Monte Carlo.

1 Introduction

Last decade was marked by the affirmation of SAR images as a tool for earth monitoring. Several studies were made confirming their relevance, where image processing techniques were developed especially devoted to them. Most of the SAR image processing techniques are based on statistical properties of the SAR data, those properties might be used for the development of tools for SAR image processing and analysis, for instance, filters to reduce speckle noise, as well as classification and segmentation algorithms.

There are many statistical models for synthetic aperture radar (SAR) images, among them, the multiplicative model is based on the assumption that the observed random field Z is the result of the product of two independent and unobserved random fields: X and Y. The random field X models the terrain backscatter and thus depends only on the type of area each pixel belongs to. The random field Y takes into account that SAR images are the result of a coherent imaging system that produces the well known phenomenon called speckle

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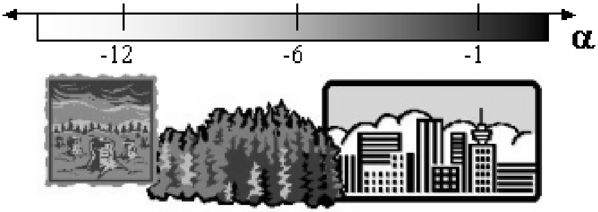

Fig. 1. Meaning of the α parameter of the $\mathcal{G}_A^0$ distribution in SAR images.

noise and are generated by performing an average of L independent image looks in order to reduce the speckle effect. This is assuming that X and Y are both weak stationary stochastic processes. The last fact is based on the assumption that the speckle noise corresponding to cells of different resolution is generated by the interaction of many independent dispersion points. Speckle refers to a noise-like characteristic produced by coherent systems, including sonar, laser, ultrasound and synthetic aperture radars. It is evident as a random structure of picture elements caused by the interference of electromagnetic waves scattered from surfaces or objects.

There are various ways of modelling the random fields X and Y. Classically, both the speckle noise Y and the backscatter X have been modelled with a $\Gamma^{1/2}$ distribution [TCG82]. This parametrization makes the return Z obey the K_A distribution. The K_A distribution fails to model many situations where the return is extremely heterogeneous, besides being computationally cumbersome.

On the other hand, in [FMYS97] was proposed the $\Gamma^{-1/2}$ distribution to model the amplitude backscatter X. This new model, when used along with the classical one for the speckle noise yields a new distribution for the return, called $\mathcal{G}_A^0$. The advantage of the $\mathcal{G}_A^0$ distribution over the classical K_A distribution is that it models very well extremely heterogeneous areas like cities, as well as moderately heterogeneous areas like forests and homogeneous areas like crops.

The $\mathcal{G}_A^0$ distribution is characterized by as many parameters as the K_A distribution: the number of looks (L), a scale parameter (γ) and a roughness parameter (α). Besides the advantages, this $\mathcal{G}_A^0$ distribution proposal has the same nice interpretational properties than the K_A distribution has, see [FMYS97]. The parameter γ is a scale parameter and is related to the relative power between reflected and incident signals. The parameter α is of particular interest in many applications, since it is directly related to the roughness of the target. The figure 1 shows how the α parameter can be used to make inferences about the type of land seen from a particular SAR image.

The figure 2 is representative of the typical complexity of real SAR images, where we can distinguish several types of roughnesses or textures. This work discusses the problem of estimating the parameters of the $\mathcal{G}_A^0$ distribution for the case of single looks that arises in image processing and analysis with large and small samples. Two typical estimation situations arise in image processing

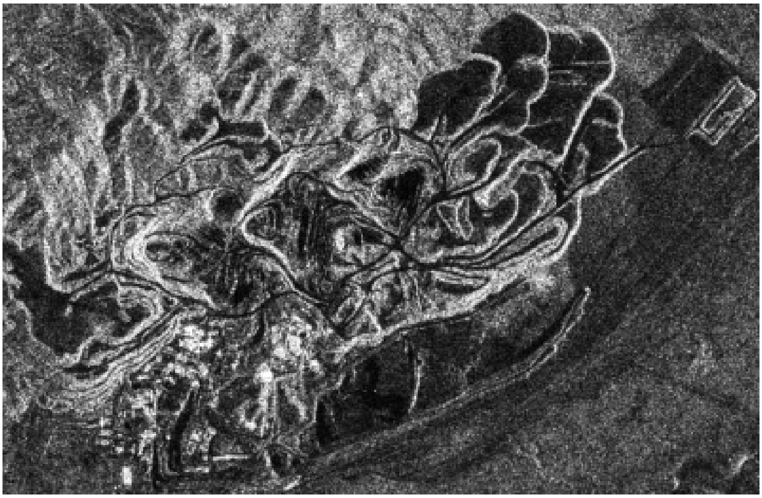

Fig. 2. SAR Image of a Chilean copper mine.

and analysis, namely large and small samples, being the latter considered in this work. Statistical inference with small samples is subjected to many problems, mainly bias, large variance and sensitivity to deviations from the hypothesized model. The last issue is also a problem when dealing with large samples.

Robustness is a desirable property for estimators, since it allows their use even in situations where the quality of the input data is below of the level accepted by standards [HRRS86]. Most image processing and analysis procedures, like classification, restoration, segmentation, use field data. A situation where this occurs is when ground controls points (GCP) appear in the SAR image, which are essential for data calibration. These points produce a return higher than the rest of the image, for this reason they are called corner reflectors. If the data from a corner reflector it is included in the SAR image, the estimation procedure is non-robust, and the results may be completely unreliable.

In Section 2 a brief explanation of the $\mathcal{G}_A^0$ distribution is presented together with the classical maximum likelihood estimators of its parameters. Section 3 presents the robust M-estimators, which are capable to deal with non perfect data. In section 4 estimation procedures are compared by means of a Monte Carlo study.

2 The $\mathcal{G}_A^0$ Distribution

The general (multilook) form of the density, which characterizes the $\mathcal{G}_A^0(\alpha, \gamma, L)$ distribution is given in [FMYS97] as

$$f(z) = \frac{2L^L \Gamma(L-\alpha)}{\gamma^\alpha \Gamma(L)\Gamma(-\alpha)} \frac{z^{2L-1}}{(\gamma + Lz^2)^{L-\alpha}}, \quad z > 0, \tag{1}$$

where $\alpha < 0$ is referred to as the roughness parameter, $\gamma > 0$ is a scale parameter and $L \geq 1$ is the number of looks. The number of looks is controlled in the early generation steps of the image, and is known beforehand or it is estimated using extended homogeneous targets. This parameter remains constant over all the image. This law was originally devised to describe extremely heterogeneous clutter, and lately proposed and assessed as an universal model for speckled imagery in [MFJB01]. Improved estimation using bootstrap for the parameters α and γ of this distribution is presented in [CFS02], while the robustness for the $L = 1$ case is studied in [BLF02] using M-estimators.

The single look case is of particular interest, and it will be considered here, since it describes the noisiest images. The distribution of interest is, then, characterized by the density

$$f(z;(\alpha,\gamma)) = -\frac{2\alpha}{\gamma^{\alpha}}\frac{z}{(\gamma+z^2)^{1-\alpha}} = -\frac{2\alpha z}{\gamma(1+z^2/\gamma)^{1-\alpha}}, \quad z > 0, \tag{2}$$

with $-\alpha, \gamma > 0$. This distribution will be denoted $\mathcal{G}_A^0(\alpha,\gamma)$, whose cumulative distribution function is given by

$$F(z;(\alpha,\gamma)) = 1 - \left(1 + z^2/\gamma\right)^{\alpha}. \tag{3}$$

Several parameter estimation techniques are available, being the most remarkable ones those based on sample moments and maximum likelihood. The k-th order moment of the $\mathcal{G}_A^0\ (\alpha,\gamma)$ distribution is given by

$$E(z^k) = \begin{cases} \gamma^{k/2}\frac{k\Gamma(k/2)\Gamma(-\alpha-k/2)}{2\Gamma(-\alpha)} & \text{if } -\alpha > k/2 \\ \infty & \text{otherwise.} \end{cases} \tag{4}$$

The maximum likelihood estimator of $\theta = (\alpha,\gamma)$, based on the observations $z_1, z_2, \ldots, z_N$, is defined as the value $\hat{\theta}_{ML}$ which maximizes $\prod_{i=1}^N f_\theta(z_i)$, or equivalently as the value $\hat{\theta}_{ML}$ which minimizes $-\sum_{i=1}^N \ln f_\theta(z_i)$. Equating to zero the derivates of this function, we get

$$\sum_{i=1}^{N} s(z_i;\theta) = 0, \tag{5}$$

where $s(z;\theta) = (s_1(z;\theta), s_2(z;\theta))^T = \frac{\partial}{\partial\theta}\ln f_\theta(z) = (\frac{\partial}{\partial\theta_1}\ln f_\theta(z), \frac{\partial}{\partial\theta_2}\ln f_\theta(z))^T$ denotes the vector of likelihood scores. Explicitly, the score functions are:

$$\begin{cases} s_1(z;\theta) = & \frac{1}{\alpha} + \ln\left(1+\frac{z^2}{\gamma}\right), \\ s_2(z;\theta) = & \frac{-\alpha}{\gamma} - \frac{1-\alpha}{\gamma-z^2}. \end{cases} \tag{6}$$

From equations (5) and (6), following [MFJB01], we derive, for the single look case, the ML-estimator $\hat{\theta}_{ML} = (\hat{\alpha}_{ML}, \hat{\gamma}_{ML})$ as:

$$\begin{cases} \hat{\alpha}_{ML} & = -\left(\frac{1}{N} \sum_{i=1}^{N} \ln \left(1 + \frac{z_i^2}{\hat{\gamma}_{ML}} \right) \right), \\ \hat{\gamma}_{ML} & = \left[\left(1 + \frac{1}{N} \sum_{i=1}^{N} \ln \left(1 + \frac{z_i^2}{\hat{\gamma}_{ML}} \right) \right) \frac{1}{N} \sum_{i=1}^{N} \left(\hat{\gamma}_{ML} + z_i^2 \right)^{-1} \right]^{-1}. \end{cases} \tag{7}$$

3 Robust Estimators

As previously seen, the parameter α of the $\mathcal{G}_A^0$ distribution is defined for negative values. For near zero values of α, the sampled area presents very heterogeneous gray values, as is the case of urban areas. As we move to less heterogeneous areas like forests, the value α diminishes, reaching its lowest values for homogeneous areas like crops. This is the reason why this parameter is regarded as a roughness or texture parameter (recall figure 1).

Corner reflectors can be considered as additive outliers in SAR imagery, as physical equipment in the sensed area that return most of the power they receive. The image in these areas is dominated by the biggest possible values admitted by the storage characteristics, and their effect is typically limited to a few pixels. Corner reflectors are either placed on purpose, for image calibration, or due to man-made objects, such as highly reflective urban areas, or the result of double-bounce reflection [OQ98].

In the reality, it is necessary to use procedures that behave fairly well under deviations from the assumed model, these procedures are called robust. One of the best known classes of robust estimators are M-estimators, which are a generalization of the ML-estimators [AGV01]. In this work, we use them to estimate the parameters of the $\mathcal{G}_A^0$ distribution. These estimators, based on a sample $z_1, z_2, \ldots, z_N$, are defined as the solution $\hat{\theta}_M$ of the estimation equation

$$\sum_{i=1}^{N} \psi(z_i; \theta) = 0. \tag{8}$$

Equation (8) is a generalization of the maximum likelihood equation (5). ψ is a composition of functions of the score function (6) and the Huber's function given by $\psi_b(y) = \min\{b, \max\{y, -b\}\}$, where b is called tuning parameter. The importance of the ψ functions is that they truncate the score of the influential observations in the likelihood equation. Many theoretical results concerning the asymptotic and the robustness properties of M-estimators are available in the literature [AGV01], [BLF02], [RV02]. On the other hand, it is possible consider M-estimators with asymmetrical influence functions [AFGP03], which depend on underlying distributions.

With the purpose of obtain unbiased and optimal estimators, we redefine the M-estimator $\hat{\theta}_M$ as a solution of the equation

$$\sum_{i=1}^{N} \psi[s(z_i; \theta) - c] = 0, \tag{9}$$

where the Fisher consistency is accomplished by means of the c function, which is defined implicitly as

$$\int_{-\infty}^{\infty} \psi[s(z_i;\theta) - c]\, \mathrm{d}F_\theta(z) = 0. \tag{10}$$

The rule for determining the tuning parameter b, is to require the asymptotic relative efficiency of the M-estimator, with respect to the ML-estimator in the model without outliers, ranges from 90% to 95% [MR96].

4 Simulation Study

A Monte-Carlo study is performed in order to assess the behavior of the robust M-estimator with respect to ML-estimator. It is considered that each sample is contaminated by a fraction ϵ of outliers of magnitude v. Hence, a sample $z_1, z_2, \ldots, z_N$ obey the following data contamination model:

$$F(z;(\alpha,\gamma);\epsilon;v) = (1-\epsilon)\, F(z;(\alpha,\gamma)) + \epsilon\delta_v(z), \tag{11}$$

where $\delta_v(z) = 1_{[v;+\infty)}(z)$ with v a very large value as compared to most of the sample data, which is chosen as a factor of the sample mean.

A numerical comparison is made over $R = 1000$ different samples generated by means of (11). Using (4), the parameter γ depends on a given value for α through $E(Z) = 1$. The methodology used to compute the estimates was that described in [MR96].

Tables 1 and 2 show, for both the ML-estimator and the M-estimator, for several values of the roughness parameter $\alpha = \{-1, -6, -10\}$, the sample mean and the mean square error, defined as $E[\hat{\alpha}] = R^{-1} \sum_{i=1}^{R} \hat{\alpha}_i$ and $mse[\hat{\alpha}] = E[\hat{\alpha} - \alpha]^2$ respectively, where α is the true value of the parameter and $\hat{\alpha}$ is its estimator. The simulation study considers the estimates in several situations, varying the sample size $N = \{9, 25, 49, 81\}$ and the contamination level $\epsilon = \{0\%, 1\%, 5\%, 10\%\}$. Also, the outliers were considered as a factor of the sample mean of $v = 15$.

The results in the tables show that both ML and M estimators exhibit almost the same behavior when the sample is exempt of contamination. Besides, when the sample size grows both methods show better estimates. Nevertheless, when the percentage of outliers increases, the ML-estimators lose accuracy faster than M-estimators. Summarizing, M-estimators show either equal or better performance than ML-estimators in all cases.

5 Conclusions

In this paper different estimators were used to estimate the roughness parameter α of the $\mathcal{G}_A^0$ distribution for the single look case. In a Monte-Carlo study, classical ML-estimators were compared with robust M-estimators, where the latter were better performance than the former in all considered situations, as varying the sample size and varying the contamination level.

Table 1. Numerical comparison of the mean between ML and M estimators, for varying α, sample size and contamination level ϵ, with $v = 15$.

		$\alpha = -1$		$\alpha = -6$		$\alpha = -10$	
ϵ	N	$E[\hat{\alpha}_{ML}]$	$E[\hat{\alpha}_M]$	$E[\hat{\alpha}_{ML}]$	$E[\hat{\alpha}_M]$	$E[\hat{\alpha}_{ML}]$	$E[\hat{\alpha}_M]$
	9	-1.162	-1.140	-6.508	-6.507	-9.997	-9.997
0%	25	-1.048	-1.041	-6.265	-6.264	-10.295	-10.295
	49	-1.013	-1.004	-6.114	-6.114	-10.175	-10.175
	81	-1.014	-1.012	-6.060	-6.060	-10.123	-10.123
	9	-0.682	-0.920	-1.818	-2.801	-2.298	-3.343
1%	25	-0.837	-0.943	-3.245	-4.355	-4.432	-5.957
	49	-0.894	-0.957	-4.042	-4.937	-5.961	-7.379
	81	-0.922	-0.967	-4.464	-5.190	-6.808	-8.036
	9	-0.668	-0.909	-1.691	-2.592	-2.130	-3.080
5%	25	-0.767	-0.900	-2.701	-3.787	-3.695	-5.080
	49	-0.796	-0.905	-3.112	-4.146	-4.286	-5.700
	81	-0.802	-0.908	-3.156	-4.183	-4.346	-5.771
	9	-0.638	-0.886	-1.553	-2.365	-1.957	-2.798
10%	25	-0.701	-0.861	-2.147	-3.111	-2.835	-3.975
	49	-0.681	-0.830	-2.136	-3.110	-2.877	-4.066
	81	-0.666	-0.814	-2.068	-3.052	-2.752	-3.941

Table 2. Numerical comparison of the mean square error between ML and M estimators, for varying α, sample size and contamination level ϵ, with $v = 15$.

		$\alpha = -1$		$\alpha = -6$		$\alpha = -10$	
ϵ	N	$mse[\hat{\alpha}_{ML}]$	$mse[\hat{\alpha}_M]$	$mse[\hat{\alpha}_{ML}]$	$mse[\hat{\alpha}_M]$	$mse[\hat{\alpha}_{ML}]$	$mse[\hat{\alpha}_M]$
	9	0.218	0.218	5.316	5.320	6.036	6.036
0%	25	0.046	0.052	1.647	1.647	3.636	3.636
	49	0.021	0.024	0.782	0.782	2.189	2.189
	81	0.014	0.016	0.444	0.444	1.415	1.415
	9	0.114	0.078	17.546	10.494	59.377	44.509
1%	25	0.045	0.042	7.815	3.225	31.439	17.195
	49	0.026	0.024	4.219	1.637	17.194	8.014
	81	0.016	0.014	2.777	1.051	11.325	4.962
	9	0.124	0.072	18.690	12.054	62.132	48.426
5%	25	0.074	0.045	11.444	5.793	40.882	26.000
	49	0.058	0.031	9.027	4.165	34.517	20.700
	81	0.050	0.022	8.690	3.835	33.603	19.612
	9	0.148	0.078	19.958	13.810	64.993	52.696
10%	25	0.110	0.052	15.480	9.404	52.624	38.525
	49	0.114	0.047	15.378	8.976	51.837	36.880
	81	0.120	0.046	15.725	9.086	53.149	37.672

As concluding remarks, one could say that the $\mathcal{G}_A^0$ distribution is a quite good model for SAR data, whose parameters have relevant and immediate physical interpretation. Estimators of these parameters can be used in various ways, for instance, as classification and segmentations tools of SAR images or development of digital filters, among others.

In future works, a simultaneous estimation of the α and γ parameters will be considered. Also, M-estimators will be studied for the multilook case.

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