Recent Algorithms for Realistic Shape from Shading

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***Abstract*—Shape from Shading is a technique that enables to compute the 3-D shape of objects from just one input image. It makes use of information on illumination and surface reflection properties. The modern model we use is given by a partial differential equation. We show how recent algorithms solving the latter give good results for real-world input images. Also, we point out that the use of parallelisation techniques for multi-core machines enable to achieve computational times of just a few seconds, even for real-world input images of several mega-pixels.**

I. Introduction

Shape from Shading (SfS) is a classic problem in computer vision. It amounts to compute at hand of just a single given grey value image the 3-D shape of depicted objects. To this end, SfS relies on information about the scene illumination and about the reflection properties of depicted objects.

SfS is usually seen as a very difficult, ill-posed problem. About ten years ago, in an important survey paper summarising the attempts up to that point, the failure to achieve useful results with SfS was acknowledged [9]: There, the authors state that SfS results are bad for synthetic input images wher all parameters are known, and that they are even worse for real-world scenes. However, we show that it is possible to achieve good results for real-world images. The method we use relies on a recent model [7] building upon the works of Prados et al. [4] and Tankus et al. [5]. Moreover, we show that the use of sophisticated numerical algorithms enables to have these results in just a few seconds of computational times, even for large input images of several mega-pixels, cf. [2], [6].

In what follows, we first summarise our model. We proceed with a description of our approaches to the 3-D reconstruction by SfS for real-world images. By this we show some results. At the end we give a conclusion.

II. The Model

Let $x \in \mathbb{R}^2$ be in the image domain Ω. Furthermore:

- $u := u(x)$ denotes the sought 3-D depth.
- $I := I(x)$ is the normalised brightness of the given grey-value image.
- f is the focal length. It denotes the distance between the optical center of the camera and the 2-D plane to which the scene of interest is mapped.

For the depth $u > 0$ holds as the depicted scene is in front of the camera, and that the distance is measured in terms of multiples of f. We will use as unknown variable $v := \ln(u)$.

The algorithms we use solve the model equation given by the *Hamilton-Jacobi partial differential equation (PDE)*

$$\frac{\mathsf{f}^2 W}{Q}\left(I - k_a I_a\right) - k_d I_d e^{-2v} - \frac{W k_s I_s e^{-2v}}{Q} R^{\alpha} = 0 \qquad (1)$$

$$\begin{aligned} \text{with} \quad W &:= \sqrt{\mathsf{f}^2 |\nabla v|^2 + (\nabla v \cdot x)^2 + Q^2} && (2) \\ Q &:= \mathsf{f} / \left(\sqrt{x^2 + y^2 + \mathsf{f}^2}\right) && (3) \\ R &:= 2Q^2/W^2 - 1 && (4) \end{aligned}$$

Thereby, $\nabla v = (v_x, v_y)^\top$ is the so-called gradient of v, and the lower indices in v_x and v_y denote partial derivatives. The surface is described here by the Phong-model [3], which is a simple surface reflection model from computer graphics. The corresponding underlying brightness relation reads as

$$I = k_a I_a + \sum_{\text{light sources}} \frac{1}{r^2}\left(k_d I_d \cos\phi + k_s I_s (\cos\theta)^{\alpha}\right) \qquad (5)$$

where I_a, I_d, and I_s are the intensities of the ambient, diffuse, and specular components of light, respectively. The constants k_a, k_d, and k_s with $k_a + k_d + k_s \leq 1$ denote the ratio of ambient, diffuse, and specular reflection. The ambient light represents light present everywhere in a given scene. The intensity of diffusely reflected light in each direction is proportional to the cosine of the angle ϕ between surface normal and light source direction. The amount of specular light reflected towards the viewer is proportional to $(\cos\theta)^{\alpha}$, where θ is the angle between the ideal mirror reflection direction of the incoming light and the viewer direction, α is a constant modelling the roughness of the material.

III. The Algorithms

There are two main roads to approach the solution of the hyperbolic PDE (1): *(i)* iterative techniques can be employed, and *(ii)* the Fast Marching (FM) method can be used. We now outline some details of corresponding algorithms.

As a *building block* for the discretisation of spatial derivatives, a stable upwind-type discretisation is used within all methods. Let h_x and h_y be the pixel widths in x- and y-direction, respectively. Denoting then by $v_{i,j}$ the value of v at the mesh point $(ih_x, jh_y)^T$, the upwind difference for the x-direction is

$$v_x(ih_x, jh_y) \approx \min\left(0, \frac{v_{i+1,j} - v_{i,j}}{h_x}, \frac{v_{i-1,j} - v_{i,j}}{h_x}\right) \quad (6)$$

Accordingly, v_y is discretised.

Iterative Algorithms

Using straight forward pointwise discretisations of all other terms in (1), easy-to-code iterative methods can be constructed solving the problem. In that context, also relatively sophisticated acceleration techniques can be applied, e.g. the fast sweeping method, or a multigrid scheme, cf. [1] for detailed descriptions.

When applying iterative techniques, one needs to engineer the following procedure in order to obtain high-quality results for SfS for real-world input images [8]:

1) Segmentation of the object of interest. This relatively sophisticated step can be realised e.g. by using active contour models.
2) Segmentation of textured parts on the object surface. This segmentation step can be achieved by relatively simple methods like e.g. an adaptive tresholding.
3) Inpainting of textured regions. This step can be realised conveniently by using diffusion models.

A result of such a proceeding is given in Figure 1.

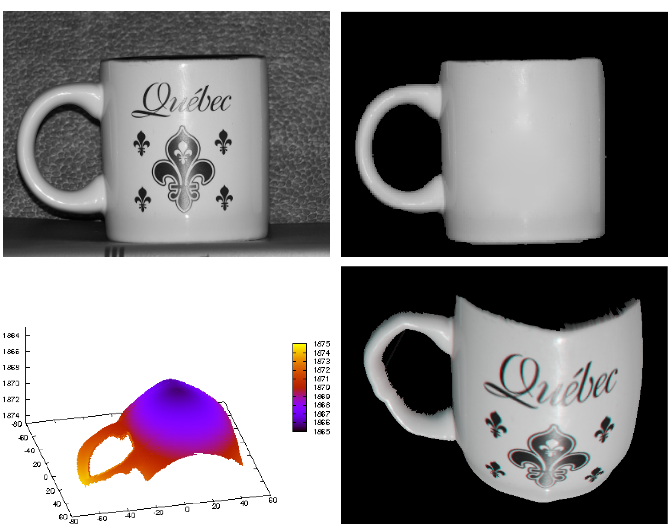

Fig. 1. Iterative methods for a real-world image. *Top row.* Original image (left) and after segmentations and inpainting (right). *Bottom row.* Plot of the computed 3-D depth (left) and rendered reconstruction (right).

Fast Marching Techniques

Assuming that it is possible to estimate the true depths at *singular points* – i.e. points on the surface of objects of interest where the 3-D depth is minimal – one can apply the FM idea. It relies on the causality principle incorporated in the underlying PDE: Beginning with the singular points, one builds up the solution surface by increasing step-by-step the depth. This results in an algorithm of complexity $O(N \log N)$, where N is the number of pixels. Moreover, novel approaches allow to parallelise the method on multi-core machines, cf. [2], [6] for details on FM for our setting and for its parallelisation.

The FM method does not need an initial segmentation as the iterative techniques do, but one needs to have good estimates of the depth at singular points. The bargain for this is that the method is efficient enough to give results for real-world images in a few seconds, cf. Figure 2 and the comments below it.

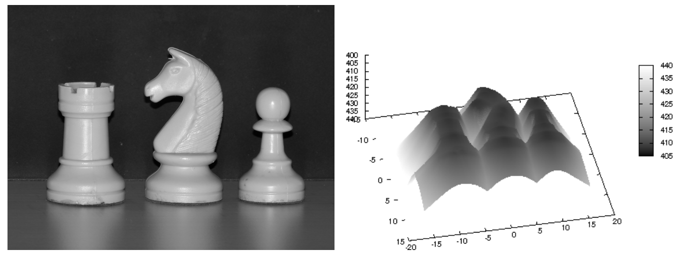

Fig. 2. FM for a real-world image. *Left.* Original image, 8 mega-pixels. *Right.* FM solution, computed with parallelisation in 35.76 seconds on a Intel Core 2 Quad Q8200, 2.33 GHz, with 2.2 MB L2 Cache and 4 GB RAM.

IV. Conclusion

We have shown that it is possible to obtain by recent models and modern algorithms good results for SfS in reasonable computing time on standard parallel hardware. Especially, we have parallelised the FM method which gives one of the fastest and most efficient algorithms in the field.

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