Numerical Algorithms for Visual Computing II

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Assignment 4 (5 Exercises) – The Öi sheet Orderings, Errors, Invariance

Exercise No. 1 – Matrix and Matrix Reloaded

Consider the method (5.3) for the approximation of the BVP for the *Poisson equation*. Let two *linear orderings* of 16 computational nodes be given as indicated by the following tables:

(a)

1	3	6	10
2	5	9	13
4	8	12	15
7	11	14	16

(b)

1	12	16	10
6	5	14	2
4	8	9	15
13	7	3	11

Determine the arising linear systems for the standard 5-point discretisation of the Laplace operator.

(3+3=6 pts)

Exercise No. 2 – Derivative Cross-Over

Consider the following discretisation of the cross derivative u_{xy}:

$$(u_{xy_C})_{i,j} \quad := \quad \frac{1}{4h^2}(U_{i+1,j+1} - U_{i-1,j+1} - U_{i+1,j-1} + U_{i-1,j-1})$$

1. Determine the local truncation error. **(2 pts)**
2. Is this discretisation isotropic or anisotropic? **(4 pts)**

Exercise No. 3 – Sobel Operator: Condition and Precision

The *Sobel operator* is a special discretisation of the first order derivative u_x:

$$(u_{x_{\text{Sobel}}})_{i,j} \quad := \quad \frac{1}{4}\begin{bmatrix}1\\2\\1\end{bmatrix} * \frac{u_{i+1,j} - u_{i-1,j}}{2h}$$

As indicated, it is derived from the central difference approximation, convolving it orthogonally to the derivative direction with a small discrete Gaussian.

1. Determine the local truncation error. **(2 pts)**
2. Is this discretisation isotropic or anisotropic? **(4 pts)**

Exercise No. 4 – Is this stencil good™ or evil™?

Consider the following stencil for the discretisation of the Laplace operator:

$$\frac{1}{4h^2}\begin{bmatrix} 1 & 0 & 1 \\ 0 & -4 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

1. Compute the local truncation error. **(2 pts)**
2. For discretising the Poisson equation, is this a method a good choice or not? Give a discussion. **(4 pts)**

Exercise No. 5 – Is this stencil good™ or evil™?

Consider the following stencil for the discretisation of the Laplace operator:

$$\alpha\begin{bmatrix} 1 & 4 & 1 \\ 4 & -20 & 4 \\ 1 & 4 & 1 \end{bmatrix}$$

1. Determine the weight α needed in order to have a *consistent* discretisation, and compute for this case the local truncation error. **(3 pts)**
2. For discretising the Poisson equation, is this a method a good choice or not? Give a discussion. **(3 pts)**