

Numerical Algorithms for Visual Computing III: Optimisation

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Assignment 8

Exercise No. 1 – Gram-Schmidt Sucks Reloaded ($3 + 3 + 3 = 9$ points)

Let the matrix A_1 be given as follows:

$$A_1 := \begin{pmatrix} 1 & 0 \\ 1 & 2 \\ 0 & 1 \\ 0 & 1 \end{pmatrix}$$

and the matrix A_2

$$A_2 := \begin{pmatrix} 8 & 21 \\ 13 & 34 \\ 21 & 55 \\ 34 & 89 \end{pmatrix}$$

Do the following task for both matrices separately:

1. Use the Gram-Schmidt algorithm for making an *orthonormal basis* out of the column vectors of A . Perform the computations with *4-digit accuracy*.
2. Check the orthogonality of the two resulting vectors q_1, q_2 from part (a). Discuss your findings.
3. Define the matrix of column vectors that you found as $Q := (q_1, q_2)$. Employ once again the Gram-Schmidt algorithm in 4-digit accuracy to orthonormalize the columns of Q . Discuss your result.

Exercise No. 2 – Gram-Schmidt Sucks Revolutions (6 points)

Compute the QR decomposition via the modified Gram-Schmidt algorithm of the matrix

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{pmatrix}.$$