# Numerical Algorithms for Visual Computing III: Optimisation

Michael Breuß and Kai Uwe Hagenburg

**Released:** 07.07.2011
**Assigned to:** Tutorial at 14.07.2011

# Assignment 7

## Exercise No. 1 – The Splitting Validation (6 points)

Prove that the Douglas-Rachford Splitting (14.18) is a valid splitting scheme.

## Exercise No. 2 – The Conjugate Convexification ($2+2+2+2=8$ points)

What is the convex conjugate function of

1. $f(x) = \exp(x)$
2. $f(x) = |x|$
3. $f(x) = \frac{1}{2}x^2$
4. $f(\vec{x}) = a^\top x - b$ with $a, b, x \in \mathbb{R}^n$

## Exercise No. 3 – Musings on Bregman Distance (6 points)

Prove the following properties of the Bregman distance. Assume $F, F_1, F_2$ convex and differentiable

1. Non-negativity $B_F(p,q) \geq 0 \quad \forall p, q$
2. Convexity: $B_F$ is convex in one or both variables.
3. Linearity: For $\lambda > 0$ it holds: $B_{F_1+\lambda F_2}(p,q) = B_{F_1}(p,q) + \lambda B_{F_2}(p,q)$

## Exercise No. 4 – The Diverging Bregman (5 points)

Compute for the convex function

$$F(p) = \sum_i p(i) \log p(i) - \sum_i p(i)$$

the Bregman distance $B_F(p,q)$.

## Exercise No. 5 – The ROF Lagrangian (5 points)

Compute the Euler-Lagrange equation of the ROF model for TV denoising in 2-D

$$\int_\Omega \|\nabla u\|_2 + \frac{\lambda}{2}\|u-f\|_2^2 \ \mathrm{d}x\, \mathrm{d}y.$$

Is the resulting PDE differentiable for all $u$? If yes, why? If not, how can you fix this?