Numerical Algorithms for Visual Computing III: Optimisation

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Assignment 6

Exercise No. 1 – Programming of First Order ($4 + 4 + 4 = 12$ points)

In the lecture we have seen methods for solving non-linear problems. Linear problems do also exist and are quite common, e.g. in economics. Given the following problem formulation: A CPU manufacturer gets silicon wafers from different sub-contractors for the production of high-end processors. Both wafers have different properties and are being used for building CPUs clocked at 1Ghz, 2GHz and 3GHz. However, both wafer types require different production techniques, so we have the following production line (per day)

	1GHz	2GHz	3GHz
Wafer "A"	20	15	5
Wafer "B"	20	3	10

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e.g. with wafer "A" it is possible to build 15 2GHz processors but only 3 with wafer "B". Furthermore, the production plan requires to build <u>at least</u> 60 1GHz processors, 15 2GHz processors and 20 3GHz processors a day. One wafer of type "A" costs 10 EUR and one of type "B" costs 7 EUR. The task is to minimize the costs under the given constraints!

1. Write the linear program of the described problem with all constraints.
2. Solve the linear program graphically with a sketch.
3. Generalize the linear program formulation for the minimization of arbitrary linear problems. Derive furthermore a dual formulation of the problem. What can you say on the duality between both problems?

Exercise No. 2 – Hanging out with Joseph Louis (6 points)

Given the problem

$$\text{minimize} \qquad f(x) = -\frac{1}{2}\sqrt{x_1} - \frac{1}{2}x_2$$

subject to

$$\begin{aligned} x_1 &\geq 0.1 \\ x_2 &\geq 0 \\ x_1 + x_2 &\leq 1 \end{aligned}$$

Solve this problem with the help of Lagrange multipliers.

Exercise No. 3 – Getting a fix (6 points)

We have seen in the lecture, that a fix-point iterative scheme arises from the augmented Lagrange method, see equation (11.18). Show that this method indeed converges according to the fixed point theorem of Banach.

Exercise No. 4 – Operation KKT (6 points)

Consider the problem

$$\text{minimize} \qquad x_1^3 + x_2^3 + x_3^3 \tag{1}$$

subject to

$$x_1^2 + x_2^2 + 3x_3 \quad \leq \quad -\frac{5}{2} \qquad \text{and} \tag{2}$$

$$x_1 + x_2 + x_3 = -2 \tag{3}$$

1. Write down the KKT conditions.
2. Use the KKT conditions to derive the dual function.
3. Do you expect a non-zero duality gap?