Simulated Annealing

A Methode to Solve Combinatorial Problems

Kurnia Hendrawan

kuhe0000@stud.uni-saarland.de

30 May 2007

Outline

Outline

What is a combinatorial problem?

- Which problems
- Parameters
- Classic examples

- given n objects to observe
- search for optimum
- Focus: "better" result

What is a combinatorial problem?

- Which problems
- Parameters
- Classic examples

- S: solutions space
- f: cost function
- f(i): quality of solution

What is a combinatorial problem?

- Which problems
- Parameters
- Classic examples

- Clique
- TSP
- Hamilton-Path

Resolution Methodes

- Numerical methode
- Heuristical methode

- "brute force"
- searching in the whole S
- pro: find the global optimum
- con: huge complexity

Resolution Methodes

- Numerical methode
- Heuristical methode

- stochastic
- local search:
- iteration with limited workspace
- searching in the neighbourhood

Local Search
How does it work?

Initilization

Choose a random solution i from the workspace (TSP: choose a random permutation for the trip)

Iteration

1. Try to find a better solution on the neighbourhood of i (TSP: change the route a bit)
2. Record better solutions
3. Iterate back
4. Terminate if the whole neighbourhood is completely searched

Local Search
How does it work?

Initilization

Choose a random solution i from the workspace (TSP: choose a random permutation for the trip)

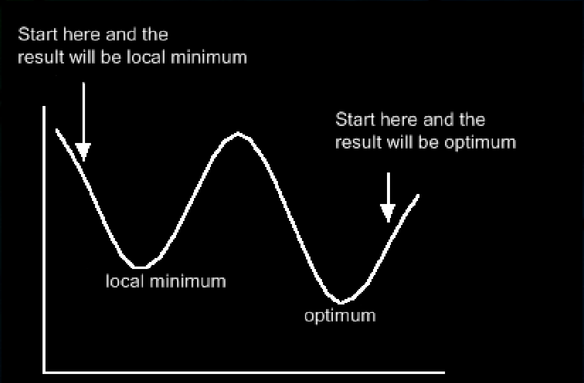
Iteration

1. Try to find a better solution on the neighbourhood of i (TSP: change the route a bit)
2. Record better solutions
3. Iterate back
4. Terminate if the whole neighbourhood is completely searched

Result

Unreliable

The result is often not the optimum one, it depends a lot on the initialization.



Weakness of local search

Problem

The algorithm only finds a local minimum

Causes

1. Only better results are accepted
2. The local minimum cannot be overcome

Improvement

How about enlarging the neighbourhood of i?

Why not?

No way!! Since otherwise it will look like numerical methodes

Other possibility

Also accept worse solutions

But..

1. How far we will accept them?
2. Which rules do we use?

Weakness of local search

Problem

The algorithm only finds a local minimum

Causes

1. Only better results are accepted
2. The local minimum cannot be overcome

Improvement

How about enlarging the neighbourhood of i?

Why not?

No way!! Since otherwise it will look like numerical methodes

Other possibility

Also accept worse solutions

But..

1. How far we will accept them?
2. Which rules do we use?

Weakness of local search

Problem

The algorithm only finds a local minimum

Causes

1. Only better results are accepted
2. The local minimum cannot be overcome

Improvement

How about enlarging the neighbourhood of i?

Why not?

No way!! Since otherwise it will look like numerical methodes

Other possibility

Also accept worse solutions

But..

1. How far we will accept them?
2. Which rules do we use?

Outline

Background

Physical Phenomene as the basis

Metropolis [1953]

Experiment on physical objects (liquids, gasses, solides) of their temperature, pressure, etc.

Annealling process of liquids

Release a minimal amount of energy

Slow annealing

Stable and steady structure (minimal energy balance)

Fast annealing

Unsteady structure, unbalance energy release (non-optimal)

Background

Physical Phenomene as the basis

Metropolis [1953]

Experiment on physical objects (liquids, gasses, solides) of their temperature, pressure, etc.

Annealling process of liquids

Release a minimal amount of energy

Slow annealing

Stable and steady structure (minimal energy balance)

Fast annealing

Unsteady structure, unbalance energy release (non-optimal)

Background

Physical Phenomene as the basis

Metropolis [1953]

Experiment on physical objects (liquids, gasses, solides) of their temperature, pressure, etc.

Annealling process of liquids

Release a minimal amount of energy

Slow annealing

Stable and steady structure (minimal energy balance)

Fast annealing

Unsteady structure, unbalance energy release (non-optimal)

Background

Physical Phenomene as the basis

Metropolis [1953]

Experiment on physical objects (liquids, gasses, solides) of their temperature, pressure, etc.

Annealling process of liquids

Release a minimal amount of energy

Slow annealing

Stable and steady structure (minimal energy balance)

Fast annealing

Unsteady structure, unbalance energy release (non-optimal)

Metropolis

Part I

Important

To overcome local minima we might also accept some worse solutions

Probability

We use a distribution which dependent on

- Temperature T (mobility of the particles)
- Energy difference $\Delta E = E_2 - E_1$

Metropolis
Part I

Important

To overcome local minima we might also accept some worse solutions

Probability

We use a distribution which dependent on

- Temperature T (mobility of the particles)
- Energy difference $\Delta E = E_2 - E_1$

Metropolis
Part II

Let p be acceptance probability

Algorithm

better solution	worse solution
$E_2 < E_1$	$E_2 > E_1$
accept ($p = 1$)	accept with $p = e^{-\frac{E_2 - E_1}{T}}$ and $p >= random[0;1]$

Metropolis
Part II

Let p be acceptance probability

Algorithm

better solution	worse solution
$E_2 < E_1$	$E_2 > E_1$
accept ($p = 1$)	accept with $p = e^{-\frac{E_2 - E_1}{T}}$ and $p >= random[0;1]$

The General Algorithm
Principle Procedure

Pseudo-code

```
INIT(i_start, T_start, HT_start);
k:=0; i:=i_start; T_k:=T_start;
repeat
 for h:=1 to HT_k
  GENERATE(j of S_i);
  if f(j)<f(i) then i:=j;
   else
    if e^(f(i)-f(j)/T_k)>random[0;1] then i:=j;
 k:=k+1;
 DETERMINE_NEIGHBOURHOOD_SIZE(HT_k);
 SCHEDULE_ANNEALING(T_k);
until End_Conditions reached;
```

OUTLINE

Parameter Selection

- Initial temperature
- Annealing schedule
- Neighbourhood H_T
- End Conditions
- Known issues

Setting

- Must be high enough to ensure result independent from initial solution

Parameter Selection

- Initial temperature
- Annealing schedule
- Neighbourhood H_T
- End Conditions
- Known issues

Setting

- geometrical: $d(T) = c \cdot T$ where $c \in [0,8; 0,99]$
- slow annealing: $d(T) = \frac{T}{1+c \cdot T}$ where $c > 0$ and small

Parameter Selection

- Initial temperature
- Annealing schedule
- Neighbourhood H_T
- End Conditions
- Known issues

Setting

- H_T must be reasonable large
- large H_T at initial stages is unnecessary since it will be "neutralized" by the initial random solution
- intensive searching towards the end of algorithm is needed to reach optimum

$\Rightarrow$ Increase H_T by temperature reduction

Parameter Selection

- Initial temperature
- Annealing schedule
- Neighbourhood H_T
- End Conditions
- Known issues

Setting

Combination of:

- reaching an end-temperature or exceeding a maximal number of iterations
- at some large interval of runtime no better solution found
- acceptance rate falls below a fixed limit (e.g. under 3%)

Parameter Selection

- Initial temperature
- Annealing schedule
- Neighbourhood H_T
- End Conditions
- Known issues

Setting

- parameter setting is the significant part to the algorithm
- SA works fine only if the parameters set correctly
- too low temperature: get stuck on a local minimum
- too high temperature: the optimum will be passed several times
- too large neighbourhood: high complexity

Outline

SUMMARY

SIMULATED ANNEALING

Meta - heuristical optimization methodes to solve combinatorial problems

BASIC

On physical process in the nature: annealing material

IDEA

Accept worse solution to gain the optimum one

EVALUATION

+ easy to implement
+ good end result (1-10% to optimum)
+ several hundreds possible application
- difficult parameter selection

Summary

Simulated Annealing

Meta - heuristical optimization methodes to solve combinatorial problems

Basic

On physical process in the nature: annealing material

Idea

Accept worse solution to gain the optimum one

Evaluation

+ easy to implement
+ good end result (1-10% to optimum)
+ several hundreds possible application
- difficult parameter selection

Summary

Simulated Annealing

Meta - heuristical optimization methodes to solve combinatorial problems

Basic

On physical process in the nature: annealing material

Idea

Accept worse solution to gain the optimum one

Evaluation

+ easy to implement
+ good end result (1-10% to optimum)
+ several hundreds possible application
- difficult parameter selection

Summary

Simulated Annealing

Meta - heuristical optimization methodes to solve combinatorial problems

Basic

On physical process in the nature: annealing material

Idea

Accept worse solution to gain the optimum one

Evaluation

+ easy to implement
+ good end result (1-10% to optimum)
+ several hundreds possible application
- difficult parameter selection

LITERATURES

- “Simulated Annealing” - im Rahmen des PS Virtual Lab” Martin Pfeiffer.
- Simulated annealing - Wikipedia, the free encyclopedia. Online. 01 May 2007. <http://en.wikipedia.org/wiki/Simulated_annealing>
- Peter Rossmanith, RWTH Aachen. Algorithmus der Woche » Simulated Annealing. Online. 05 May 2007 <http://www-i1.informatik.rwth-aachen.de/~algorithmus/algo41.php>