Mathematical Foundations of Computer Vision

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Assignment 6 – Completing the Sixpack

Exercise No. 1 – Exponentiate Rodrigues

The aim of this exercise is to validate the *formula of Rodrigues* for a rotation matrix: Given $\vec{w} \in \mathbb{R}^3$, the matrix exponential $R = e^{\hat{w}}$ is given by

$$e^{\hat{w}} \;=\; I + \frac{\hat{w}}{\|\vec{w}\|} \sin \|\vec{w}\| + \frac{\hat{w}^2}{\|\vec{w}\|^2} \left(1 - \cos \|\vec{w}\|\right) \tag{1}$$

The proof is done in three steps:

(a) Validate for $\vec{w}$ being of unit length, that *(i)* $\hat{w}^2 = \vec{w}\vec{w}^\top - I$ and *(ii)* $\hat{w}^3 = -\hat{w}$. **(4pts)**

(b) Show that it holds:

$$e^{\hat{w}t} \;=\; I + \left(t - \frac{t^3}{3!} + \frac{t^5}{5!} \mp \cdots\right)\hat{w} + \left(\frac{t^2}{2!} - \frac{t^4}{4!} + \frac{t^6}{6!} \mp \cdots\right)\hat{w}^2$$

(6pts)

(c) Prove that (1) holds true. **(6pts)**

Exercise No. 2 – Special Orthogonal Logarithms

Let us consider

Theorem 1 *Given any $R \in SO(3)$, there exists a (in general, not unique) vector $v \in \mathbb{R}^3$ such that $R = e^{\hat{v}}$. We denote the inverse of the exponential map as $\hat{v} = \log(R)$.*

Prove the theorem. **(6pts)**

Exercise No. 3 – True Lies

Consider twist coordinates ξ. Prove or disprove for $\vec{w} \neq \vec{0}$

$$e^{\hat{\xi}} \;=\; \begin{pmatrix} e^{\hat{w}} & \frac{(I - e^{\hat{w}})\hat{w}v + \vec{w}\vec{w}^\top v}{\|w\|} \\ \vec{0}^\top & 1 \end{pmatrix} \tag{2}$$

where $v(t) = \dot{T}(t) - \hat{w}T(t)$. **(8pts)**