Mathematical Foundations of Computer Vision

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Assignment 1 – Good ol' Euclidean stew

Three small exercises as an Hors d'œuvre for the course.

In this assigment, we consider three candidates for bases of the good ol' 3-D Euclidean space:

$$B_1 := [e_1, e_2, e_3] \;, \quad B_2 := \begin{pmatrix} 1 & 1 & 0 \\ 3 & 0 & 2 \\ 0 & 1 & 3 \end{pmatrix} \quad \text{and} \quad B_3 := \begin{pmatrix} 1 & 1 & 2 \\ 3 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix} \tag{1}$$

Exercise No. 1 – Buying the baking ingredients

Use the definition of linear dependence/independence in order to answer the following questions:

(a) Is B_2 a basis? **(3pts)**

(b) Is B_3 a basis? **(3pts)**

In addition:

(c) For those matrices that do not represent a basis, state the subspace which is spanned by the vectors. **(3pts)**

(d) Compute the volume contained in the parallelepipedon spanned by the column vectors of B_2. (Hint: One may use the determinant.) **(3pts)**

Exercise No. 2 – Stewing the bases

Having tasty ingredients on our table, we now look for ways to transform them:

(a) Determine the basis transformation from B_1 to B_2. **(4pts)**

(b) Determine the basis transformation from B_2 to B_1. **(6pts)**

Exercise No. 3 – Pureeing the grains

We now deal with the smallest parts of our stew:

(a) Transform the point

$$u := \begin{pmatrix} 22 \\ 55 \\ 11 \end{pmatrix} \quad \text{in the Cartesian basis } B_1 \tag{2}$$

into new coordinates w.r.t. B_2. **(4pts)**

(b) Transform the point

$$w := \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \quad \text{given in the basis } B_2 \tag{3}$$

to Cartesian coordinates. **(4pts)**