What is the Range of Surface Reconstructions from a Gradient Field?

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(ECCV 2006)

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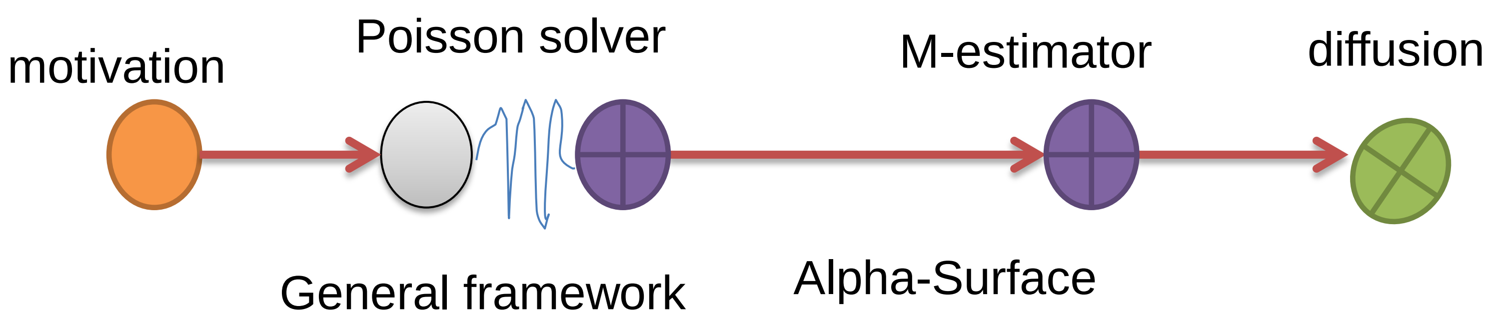
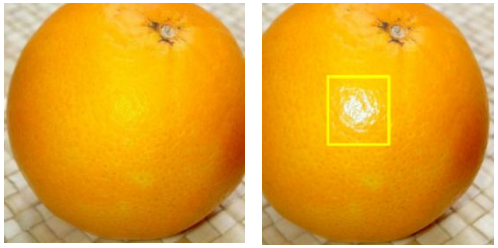
Motivation


motivation
Poisson solver
M-estimator
diffusion
General framework
Alpha-Surface

Gradient field and its application

- Compute gradient of image
- Manipulate the gradient field in order to achieve the desired goal

Texture Flattening



Selection Editing

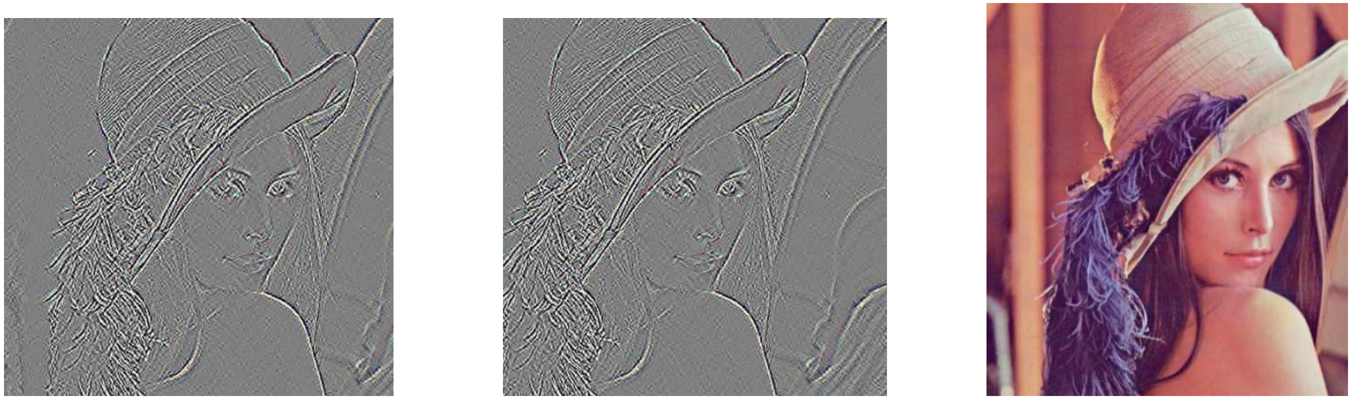
Seamless Cloning

Integrating the Modified Gradient Field

In order to integrate the gradient field it should be curl-free:

$$0 = curl\,(\nabla f(x)) = curl\,(f_x(x), f_y(x))^T = f_{yx}(x) - f_{xy}(x) \Leftrightarrow$$

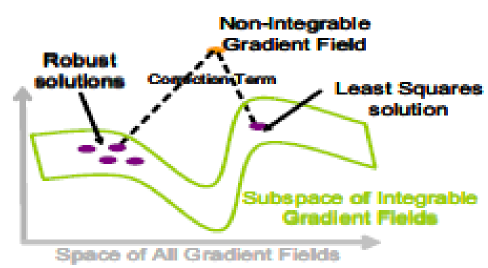
$$f_{yx}(x) = f_{xy}(x)$$

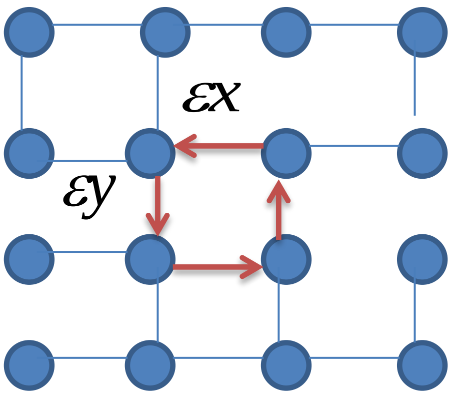


Second derivatives: f_{xy} and f_{yx} . They are identical! Right: Integration of gradient field (f_{xy} , f_{yx}) which is identical to original image.

Integrating the Modified Gradient Field

- In fact, the modified gradient field might even be non-integrable!





Left: space of all solution gradient field to make it integrable. right: add $(\varepsilon x, \varepsilon y)$ add correction

Problem statement

- A common approach to achieve the surface from the non-integrable gradient field is to minimized the last square error function:

$$J(Z) = \iint ((Z_x - p)^2 + (Z_y - q)^2)dxdy$$

- The goal is to obtain surface Z. p(x,y) and q(x,y) are given non-integrable gradient field.

Problem statement

- Which can also write as:

$$\begin{pmatrix} Z_x \\ Z_y \end{pmatrix} = \begin{pmatrix} p \\ q \end{pmatrix} + \begin{pmatrix} \varepsilon x \\ \varepsilon y \end{pmatrix}$$

- The Euler-Lagrange equation gives the Poisson equation:

$$\nabla^2 Z = div \begin{pmatrix} p \\ q \end{pmatrix}$$

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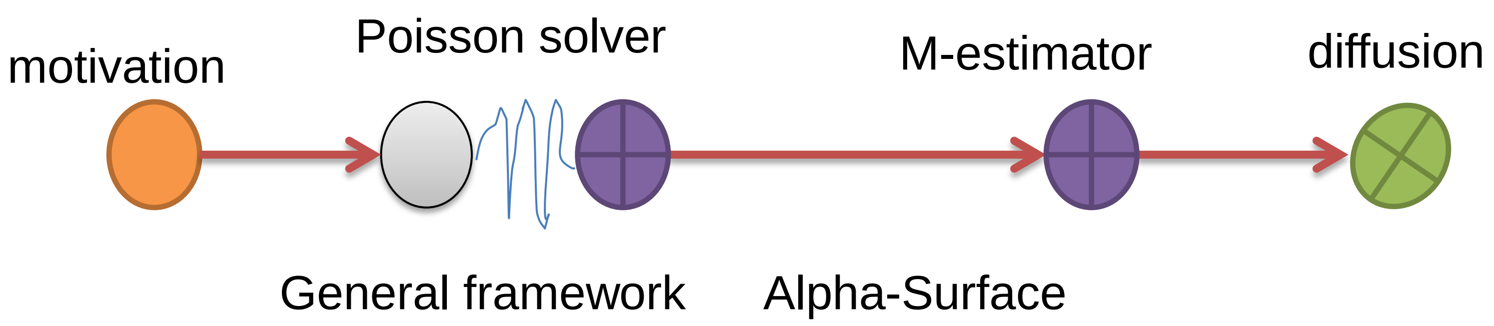
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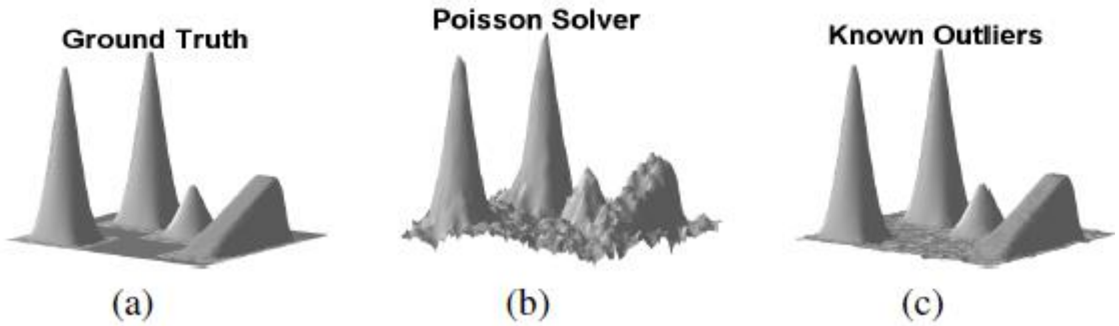
Problem of Poisson equation

- Least square solution doesn't perform well in presence of outliers:



Effect of outliers in 2D integration (a) True surface (b) Reconstruction using Poisson solver. (c) If the location of outliers were known, rest of the gradients can be integrated to obtain a much better estimate.

General framework

A general solution can be obtained by minimizing the following n-th order error functional:

$$J = \iint E(Z, p, q, Z_{x^a y^b}, p_{x^c y^d}, q_{x^c y^d}, \ldots)\, dxdy$$

$$a + b = k,\ c + d = k - 1 \quad \text{for some positive integer } k,$$

$$Z_{x^a y^b} = \frac{\partial^k Z}{\partial x^a \partial y^b},\quad p_{x^c y^d} = \frac{\partial^{k-1} p}{\partial x^c \partial y^d},\quad q_{x^c y^d} = \frac{\partial^{k-1} q}{\partial x^c \partial y^d}$$

$$1 \le k \le n; \qquad\qquad if\ k = 1$$

$$J = \iint E(Z, p, q, Z_x, Z_y)\, dxdy$$

General framework

the Euler - Lagrange equation gives :

$$\frac{\partial E}{\partial Z} = div\left(\frac{\partial E}{\partial Z_x}, \frac{\partial E}{\partial Z_y}\right) \quad (1)$$

if we consider following form for $\frac{\partial E}{\partial Z_x}, \frac{\partial E}{\partial Z_y}$:

$$\frac{\partial E}{\partial Z_x} = f_1(Z_x, Z_y) - f_3(p,q),$$
$$\frac{\partial E}{\partial Z_y} = f_2(Z_x, Z_y) - f_4(p,q) \quad (2)$$

General framework

while the modified gradient field is curl free by substituiting (2) in to(1):

$$div(f_1(Z_x,Z_y), f_2(Z_x,Z_y)) - \frac{\partial E}{\partial Z} = div(f_3(p,q), f_4(p,q))$$

In all solutions we assume Neumann boundary conditions given by:

$$\nabla Z.\hat{n} = 0$$

Poisson solver

- To achieve Poisson equation from the general solution its just need to assume:

$$\nabla^2 Z = div(p, q)$$

	$\frac{\partial E}{\partial Z} = 0$	$f_1(Z_x, Z_y)$	$f_2(Z_x, Z_y)$	$f_3(p,q),$	$f_4(p,q),$
		Z_x	Z_y	p	q

$$div(f_1(Z_x, Z_y), f_2(Z_x, Z_y)) - \frac{\partial E}{\partial Z} = div(f_3(p,q), f_4(p,q))$$

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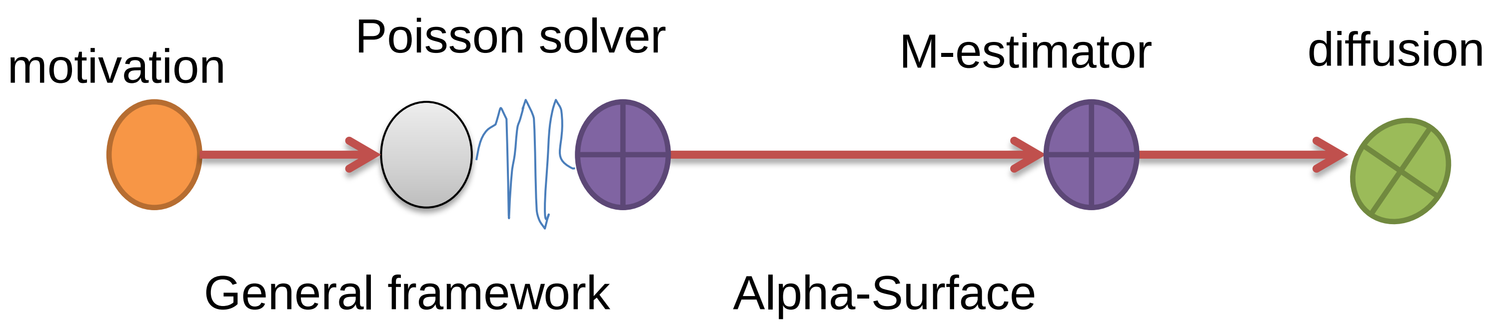
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Continuum of solution

- Techniques for robust estimation:

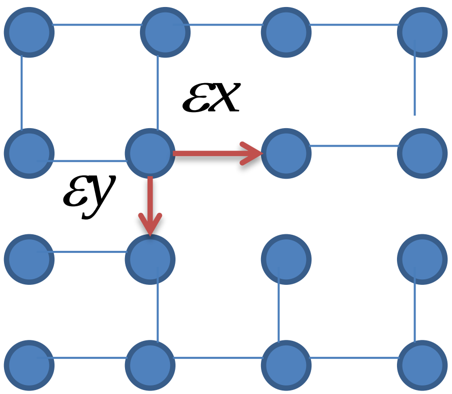
1. α- Surface: Anisotropic scaling using binary weights
2. Anisotropic scaling using continuous weight
3. Affine transformation of gradient using diffusion tensor

α- Surface

- Define initial spanning tree which is all gradient correspond to edge and are inliers

$$|\varepsilon x| = |Z_x - p| \le \alpha$$

$$|\varepsilon y| = |Z_y - q| \le \alpha$$



If α=0 we get our initial spanning tree and if α=1 we will get our poisson solver.

By changing α one can trace a path in the solution space.

α- Surface formulation

The α- Surface is a weighted approach where the weight are 1 for gradients in S and otherwise zero.

$$b_x(x,y)=1 \text{ if } Z_x \in S,\ 0 \text{ o.w.}, \quad b_y(x,y)=1 \text{ if } Z_y \in S,\ 0 \text{ o.w.},$$

$$J(Z) = \iint b_x(Z_x - p)^2 + b_y(Z_y - q)^2\,dxdy$$

Corresponding Euler_ Lagrange is:

$$div(b_x Z_x, b_y Z_y) = div(b_x p, b_y q)$$

Anisotropic scaling using continuous weight

- M- estimator: the effect of outliers is reduced by replacing the squared error residual by another function of residual:

$$J(Z) = \iint w(\varepsilon_x^{k-1})(Z_x - p)^2 + w(\varepsilon_y^{k-1})(Z_y - q)^2 dxdy$$

Affine transformation of gradient using diffusion tensor

- A method for image restoration from noisy image.

$$E(Z) = \int \left\| D\left(\begin{pmatrix} Z_x \\ Z_y \end{pmatrix} - \begin{pmatrix} p \\ q \end{pmatrix}\right) \right\|^2 dxdy$$

- The Euler-Lagrange gives:

$$div(D.\nabla Z) = div\left(D\begin{pmatrix} p \\ q \end{pmatrix}\right)$$

Affine transformation of gradient using diffusion tensor

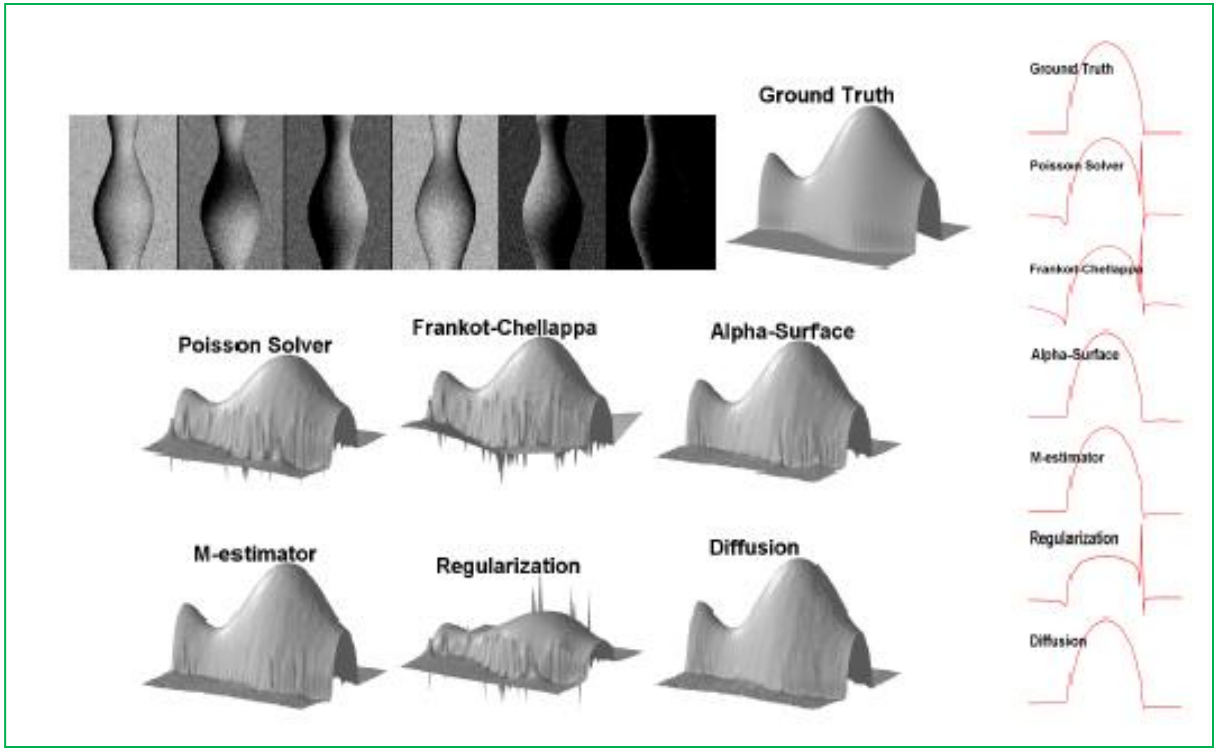
D is 2×2 symmetric , positive-definite matrix at each pixel.

$$D = \begin{pmatrix} d_{11}(x,y) & d_{12}(x,y) \\ d_{21}(x,y) & d_{22}(x,y) \end{pmatrix}$$

The gradients are scaled and lineary combined as below :

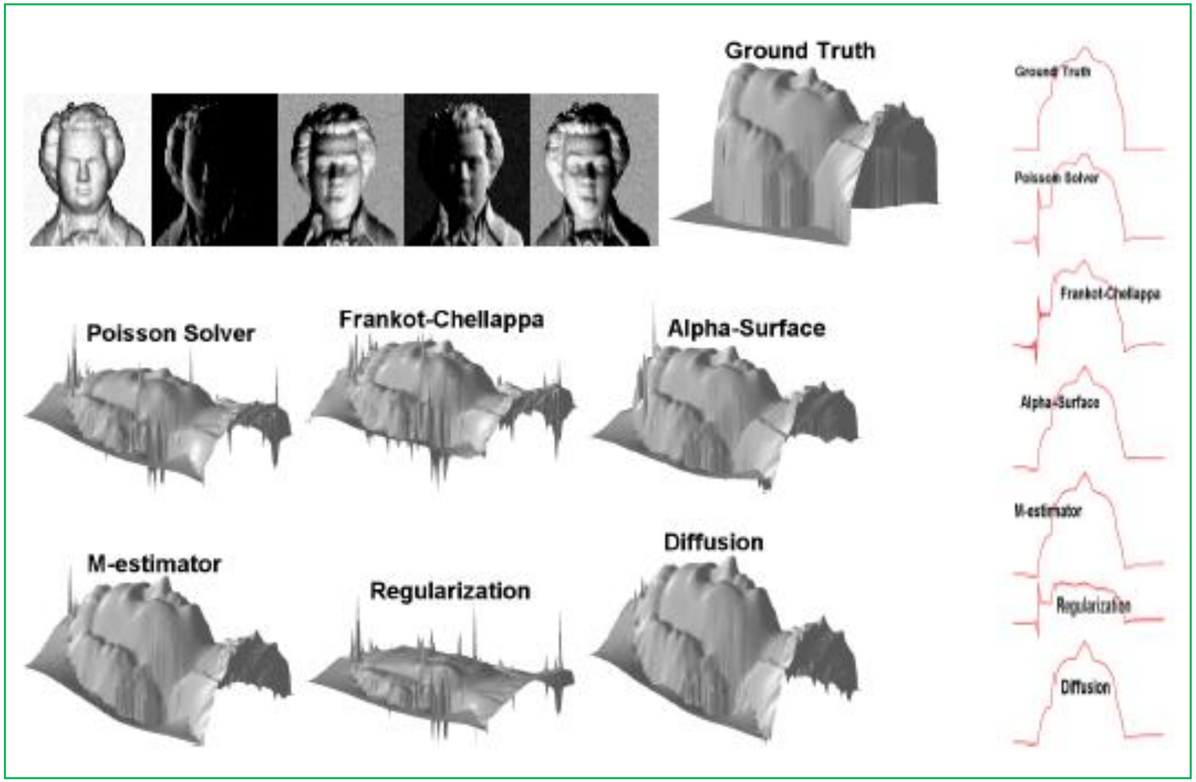
$$div\begin{pmatrix} d_{11}Z_x + d_{12}Z_y \\ d_{21}Z_x + d_{22}Z_y \end{pmatrix} = div\begin{pmatrix} d_{11}p + d_{12}q \\ d_{21}p + d_{22}q \end{pmatrix}$$

results



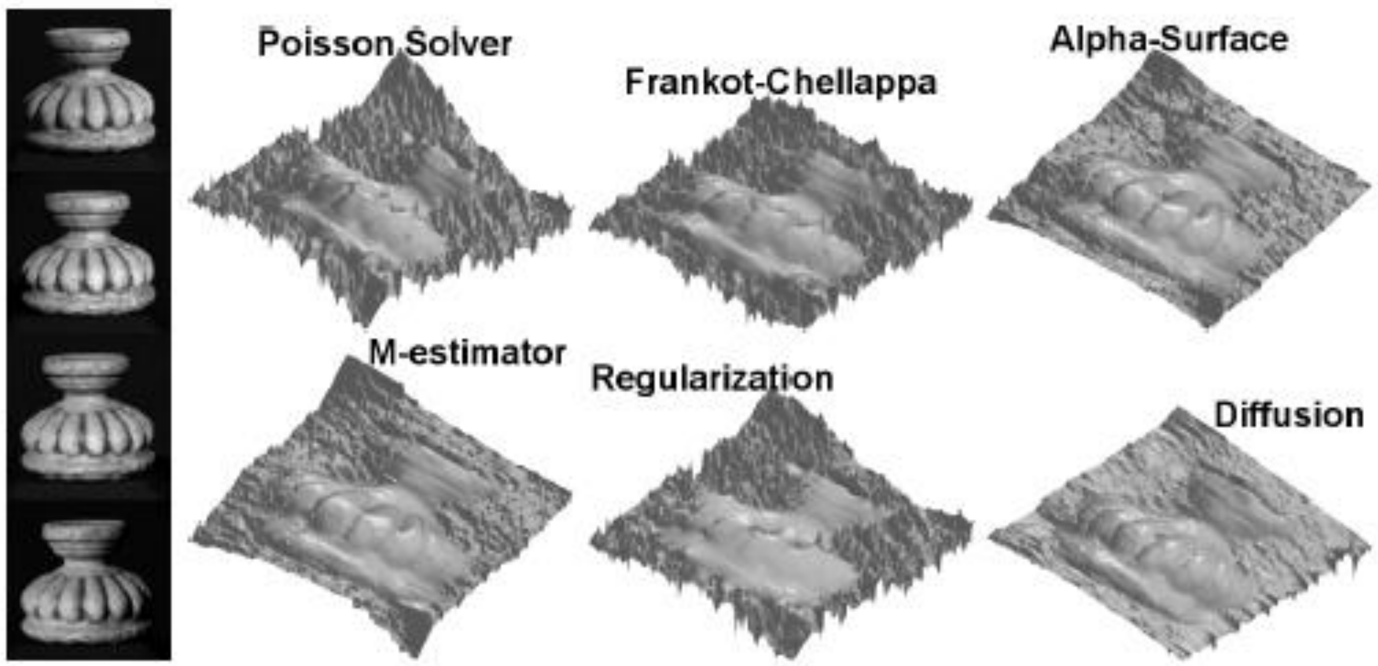
Photometric Stereo on Vase: (Top row) Noisy input images and true surface (Next two rows) Reconstructed surfaces using various algorithms. (Right Column) One-D height plots for a can line across the middle of Vase. Better results are obtained using α-surface, Diffusion and M-estimator as compared to Poisson solver, FC and Regularization

results



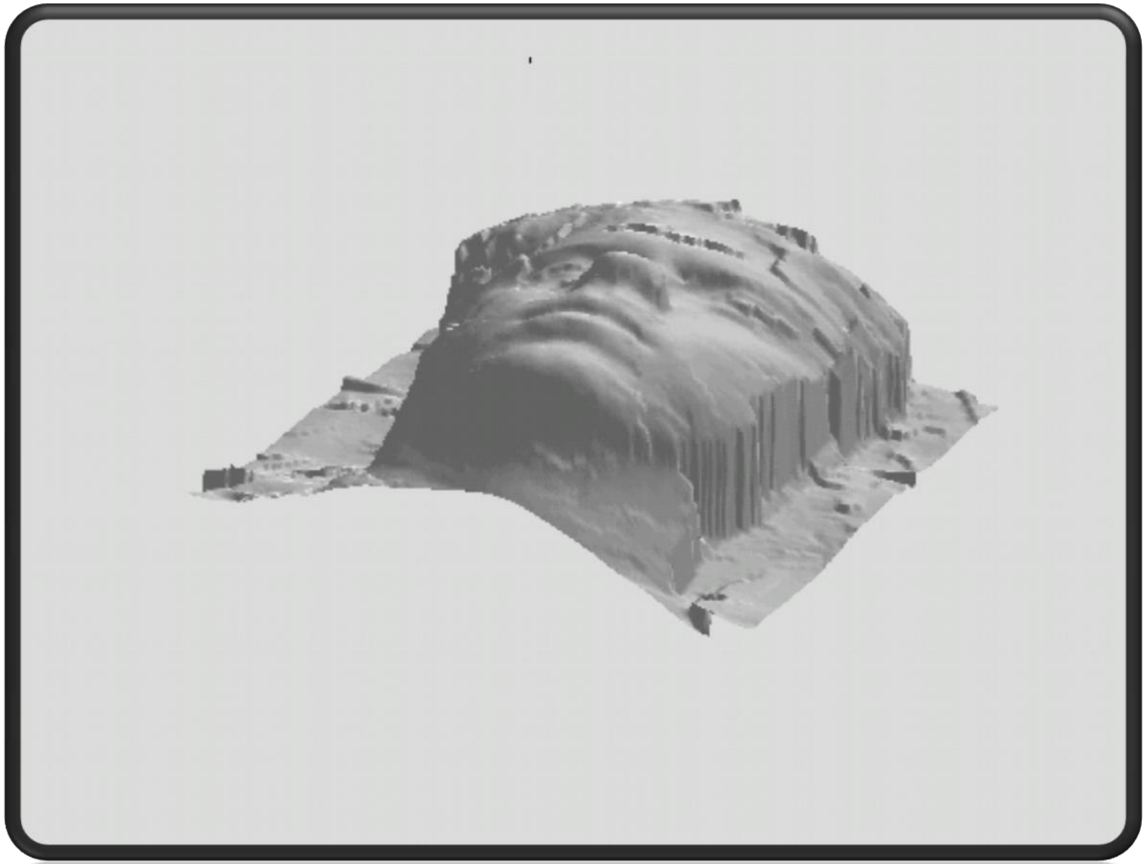
Photometric Stereo on Mozart: Top row shows noisy input images and the true surface. Next two rows show the reconstructed surfaces using various algorithms. (Right Column) One-D height plots for a scan line across the Mozart face. Notice that all the features of the face are preserved in the solution given by α-surface, Diffusion and M-estimator as compared to other algorithms.

results

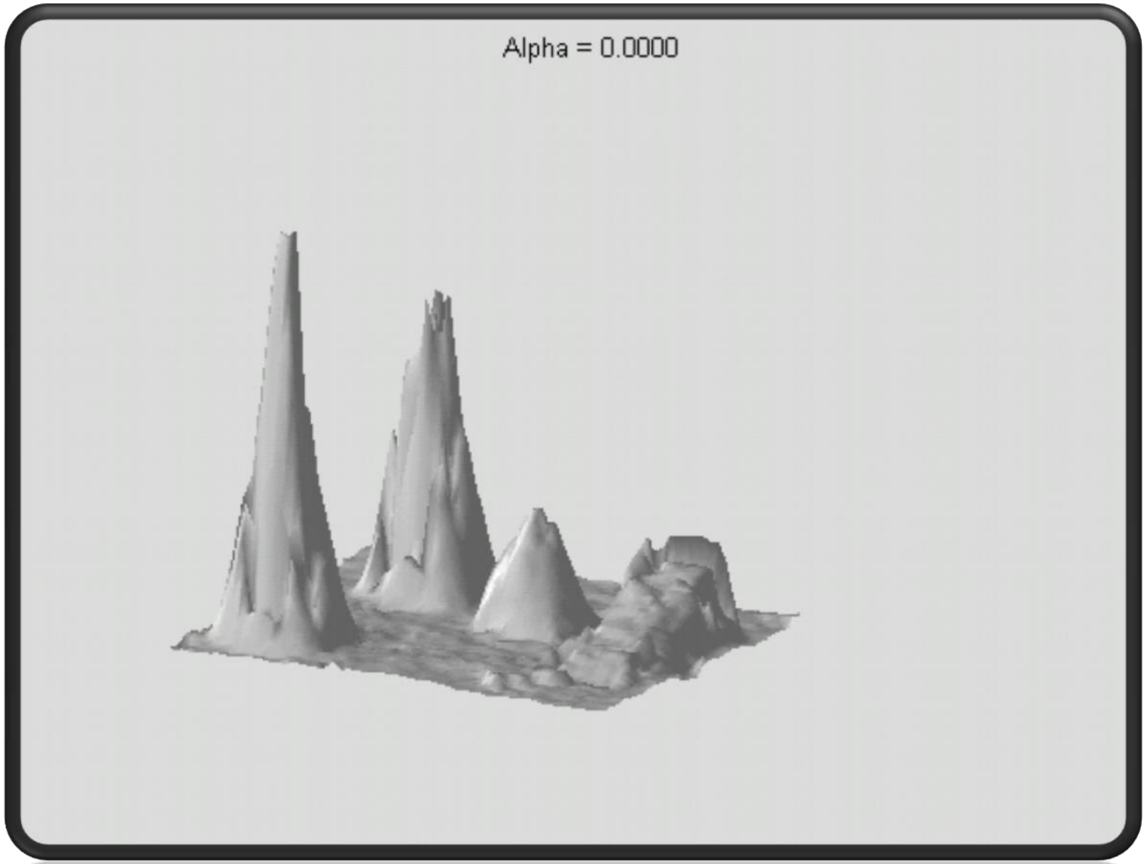


Photometric Stereo on Flowerpot: Left column shows 4 real images of a flower pot. Right columns show the reconstructed surfaces using various algorithms. The reconstructions using Poisson solver and Frankot-Chellappa algorithm are noisy and all features (such as top of flower pot) are not recovered. Diffusion, α-surface and M-estimator methods discount noise while recovering all the salient features.

results



results



reference

- A. Agrawal, R. Raskar: Gradient Domain Manipulation Techniques in Vision and Graphices. ICCV 2007 Course
- Advanced Image Analysis, Lecture 9 by Dr. Christian Schmaltz
- http://www.cfar.umd.edu/~aagrawal/eccv06/RangeofSurfaceReconstructions.html

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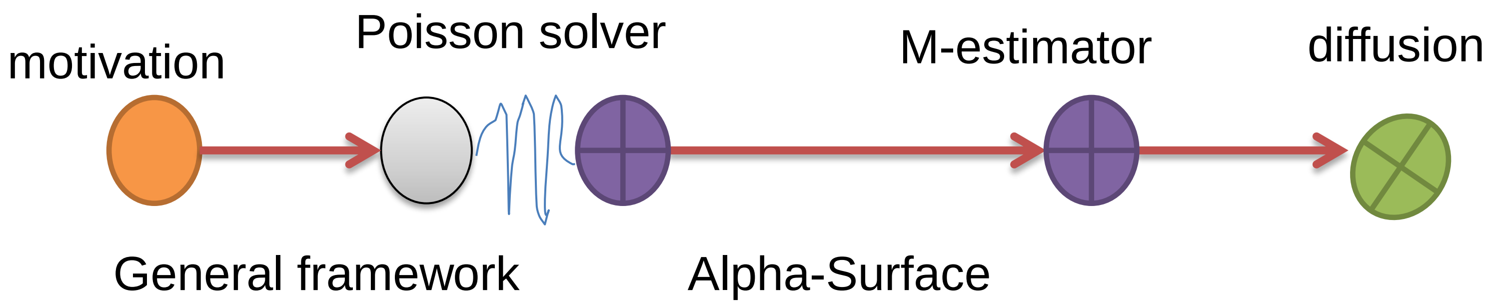
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Question?

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