Improving the Robustness

of

Variational Optical Flow

through

Tensor Voting

by: Hatem A. Rashwan, Domenec Puig, Miguel Angel Garcia

presented by:

Merlin Lang

langmerlin@stud.uni-sb.de

Milestones And Advatages in Image Analysis

Mathematical **I**mage **A**nalysis Group

Saarland University

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Motivation

- Variational methods outperform other methods
- State of the art method: complementary optic flow
- Improvement with tensor voting

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Complementary Optic Flow Model

- Given and image sequence $f(\mathbf{x})$ with $\mathbf{x} := (x, y, t)$ and displacement $\mathbf{w} = (u, v, 1)$
- Energy functional formulation:

$$E(\mathbf{w}) = \int_\Omega (\underbrace{M(\mathbf{w}, f)}_{\text{data term}} + \underbrace{\alpha V(\nabla_2 u, \nabla_2 v, f)}_{\text{smoothness term}}) \, dxdy$$

- Minimization with Euler-Lagrange-Equations:

$$0 = \partial_u M - \alpha(\partial_x(\partial_{u_x} V) + \partial_y(\partial_{u_y} V))$$
$$0 = \partial_v M - \alpha(\partial_x(\partial_{v_x} V) + \partial_y(\partial_{v_y} V))$$

Data Term

- Given grey value constancy

$$f(\mathbf{x} + \mathbf{w}) = f(\mathbf{x})$$

- can be linearized as

$$f_x u + f_y v + f_t = \mathbf{w}^T \nabla_3 f = 0$$

- Rewriting to a least squares data term

$$\begin{aligned} M &= (\mathbf{w}^T \nabla_3 f)^2 \\ &= \mathbf{w}^T \nabla_3 f (\nabla_3 f)^T \mathbf{w} \\ &= \mathbf{w}^T J_0 \mathbf{w} \end{aligned}$$

- Where J_0 is called the motion tensor

- J_0 unsufficient since aperture problem present
- Remedy: Gradient constancy

$$\nabla_3 f(\mathbf{x}+\mathbf{w}) = \nabla_3 f(\mathbf{x})$$

- One can use the final Motion Tensor

$$J = \nabla_3 f (\nabla_3 f)^T + \gamma(\nabla_3 f_x (\nabla_3 f_x)^T + \nabla_3 f_y (\nabla_3 f_y)^T)$$

- With postponing the linearisation:

$$\begin{aligned} M(u,v) =& \Psi_M((f(\mathbf{x}+\mathbf{w}) - f(\mathbf{x}))^2) \\ &+ \gamma\Psi_M((\nabla_2 f(\mathbf{x}+\mathbf{w}) - \nabla_2 f(\mathbf{x}))^2) \end{aligned}$$

- Using the robust penalizer

$$\Psi_M(s^2) = \sqrt{s^2+\xi^2}$$

Smoothness Term

- Classical homogenious regularisation

$$\begin{aligned} V(\nabla_2 u, \nabla_2 v) &= |\nabla_2 u|^2 + |\nabla_2 v^2| \\ &= (u_x^2 + u_y^2) + (v_x^2 + v_y^2) \end{aligned}$$

- Compute eigenvectors of structure tensor

$$S_\rho = K_\rho * (\nabla_2 f \nabla_2 f^T)$$

- Results in joint image- and flow-driven regularisation

$$V(\nabla_2 u, \nabla_2 v) = (e_1 \nabla_2 u)^2 + (e_2 \nabla_2 u)^2 + (e_1 \nabla_2 v)^2 + (e_1 \nabla_2 v)^2$$

- Yields the rubustified smoothness term

$$\begin{aligned} V(\nabla_2 u, \nabla_2 v) = &\Psi_V((s_1^T \nabla_2 u)^2) + (s_1^T \nabla_2 v)^2) \\ &+\Psi_V((s_2^T \nabla_2 u)^2) + (s_2^T \nabla_2 v)^2) \end{aligned}$$

- Results in new Euler Lagrange Equations:

$$0 = \partial_u M - \alpha(div D_u(s_1, s_2, \nabla_2 u)\nabla_2 u)$$
$$0 = \partial_v M - \alpha(div D_v(s_1, s_2, \nabla_2 v)\nabla_2 v)$$

- with

$$D_p(s_1, s_2, \nabla_2 p) = (s_1, s_2) \begin{pmatrix} \Psi'_V((s_1^T \nabla_2 p)^2) & 0 \\ 0 & \Psi'_V((s_2^T \nabla_2 p)^2) \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \end{pmatrix}$$

- called the “diffusion tensor”

Constraint Adaptive Regularizer (CAR)

- Regularisation Tensor.

$$R_p = K_\rho * \left(\nabla_2 f (\nabla_2 f)^T + \gamma \left(\nabla_2 f_x (\nabla_2 f_x)^T + \nabla_2 f_y (\nabla_2 f_y)^T\right)\right)$$

- Single Robust Penalisation.

$$\begin{aligned} V(\nabla_2 u, \nabla_2 v) = \Psi_V ((r_1^T \nabla_2 u)^2) + (r_1^T \nabla_2 v)^2) \\ +(r_2^T \nabla_2 u)^2) + (r_2^T \nabla_2 v)^2) \end{aligned}$$

- gives final diffusion tensor

$$D_p(s_1, s_2, \nabla_2 p) = (r_1, r_2) \begin{pmatrix} \Psi'_V ((r_1^T \nabla_2 u)^2) + r_1^T \nabla_2 v)^2) & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \end{pmatrix}$$

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Pre-segmentation of image pixels

Homogeneous and textured regions

- Compute signal to noise ratio

$$SNR = 20log_{10}(\mu/\sigma)$$

- Classify as homogenious if $SNR > \tau$ and

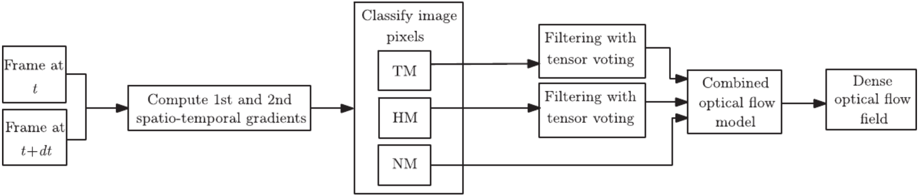
$$cos(\beta) = \frac{1}{\sqrt{1 + ||\nabla_3 f||}} \approx 0$$

- else classify as texture moving if $SNR \leq \tau$, above holds and

$$cos(\delta) = \frac{f_t}{||\nabla_3 f|| + \epsilon} \approx 1$$

- else as non moving

Approach overview



Overview of the model using tensor voting

Tensor Voting

- Tensor Voting for pixel p:

$$TV(p) = \sum_{q \in \Theta(p)} SV(v, S_q) + PV(v, P_q) + BV(v, B_q)$$

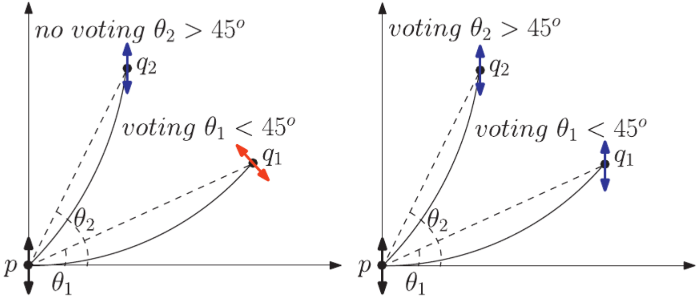
- Where SV stick, PV plate and BV ball tensor votes
- Stick voting by rotation oround surface normal and applying

$$f(\Theta) = \begin{cases} exp\left(\frac{l(\Theta)+bk(\Theta)}{\sigma}\right) & if\ \Theta \leq \frac{\pi}{4} \\ 0 & else \end{cases}$$

- BV and PV obtained by integration

Smoothing image gradients

- Apply Tensor Voting after segmentation to TM and HM pixels
- Only applied to the same class of pixels
- No voting for pixels with huge gradient difference



(a) (b)

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Adapted optical flow model

- Replace Gaussion Convolution with TV

$$T = TV(\nabla_3 f) + \gamma(TV(\nabla_3 f_x) + TV(\nabla_3 f_y))$$

- Change CAR to:

$$R = TV(\nabla_2 f) + \gamma(TV(\nabla_2 f_x) + TV(\nabla_2 f_y))$$

- With additional regularisation

$$M(\mathbf{w}, f) = \mathbf{w}^T T \mathbf{w}$$

$$V(\nabla_2 u, \nabla_2 v) = \Psi_V(R) + R$$

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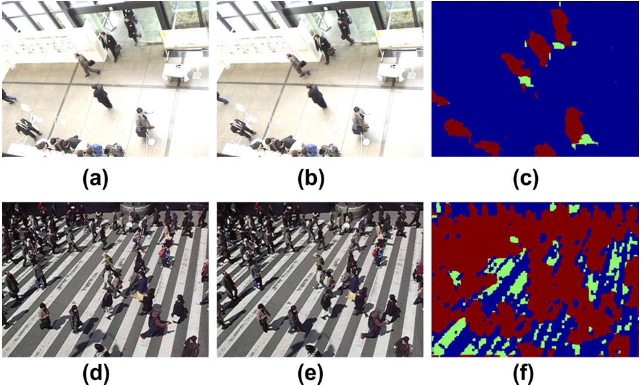
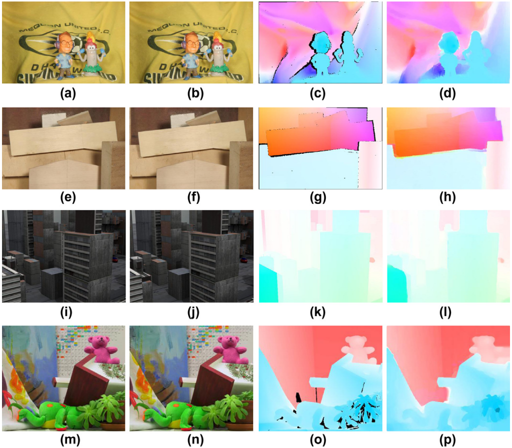
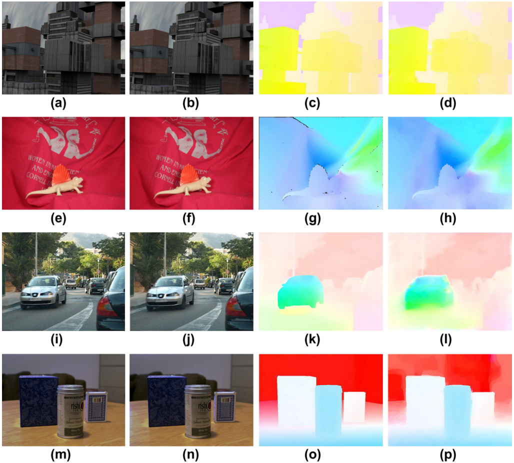
Experiment and Results



(a)Frame at time t in sequence OPEN-HOTEL.(b) Frame at time t + dt. (c) Classified pixels: red pixels are textured-moving regions, green pixels are homogeneous- moving regions and blue pixels are stationary (not moving) regions. (d) Frame at time t in sequence STREET-CROSS. (e) Frame at time t + dt. (f) Classified pixels: red pixels are textured-moving regions, green pixels are homogeneous-moving regions and blue pixels are stationary (not moving) regions.



Results for some Middlebury sequences with corresponding ground-truth. (1st column and 2nd column) Frames 10 and 11. (3rd column) Ground-truths (black points correspond to pixels without available ground-truth). (4th column) Optical flow fields obtained with the proposed approach.



Results for some Middlebury and MIT sequences with associated ground-truths. (1st column and 2nd column) Two consecutive frames. (3rd column) Ground-truths. (4th Column) Optical flow fields obtained with the proposed approach.

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Summary

- Proposed method enhances Complementary model with Tensor voting
- Separately applied to homogeneous-moving and textured-moving regions
- Proposed model yields flow fields with lower quantitative errors
- Drawback: Computational Complexity

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Thank you for your attention!