Smooth Local Histograms Filters

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Overview

Motivation

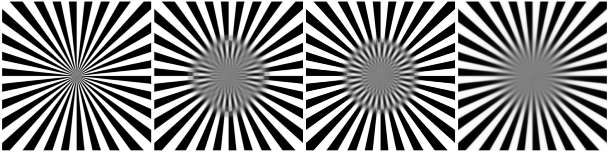
- An image histogram species how often a gray value appears within an image. It does not contain spatial information.
- Local histograms maps the tonal distribution within an image neighborhood.
- Used in many computer vision and image processions operations:
  - median filter
  - dilation (0%) and erosion (100%)
  - bilateral filter
  - mean shift
  - histogram equalization

Cons: Local histograms are expensive over large neighborhoods.
Naively implemented, the cost of construction is $O\left(n^2 \cdot \log n\right)$ for sorted histograms, and $O\left(n^2\right)$ for a binned histograms, where n is neighborhood size.

In this paper, the authors demonstrate a method for constructing local histogram in constant time regardless of neighborhood size.

Accelerating the Median Filter

- Effort have been made by various people to accelerate local histograms:
  - Huang [1975] - incrementally calculating histograms with $O(n)$ for rectangular neighborhoods.
  - Weiss [2006] - $O(\log(n))$
  - Porikli [2005], Perreault and Herbert [2007] - constant time.
  - Many other algorithms for accelerating various histogram-based filters.
- However, these algorithms are not isotropic, do not use a smoothed histogram and give rise halos, gradient reversal, and other artifacts.



Left to right: Pinwheel image, Photoshop Median Filter, Isotropic Equal Weight Median, Authors' Median Filter.

Definition

The smooth local histogram can be thought of as a *kernel density estimator* (also called Parzen-Rosenblatt window estimator).

$$f_p(s) = \frac{1}{n} \sum_{i=1}^{n} K\left(I_{q_i} - s\right)$$

- K is the smoothing kernel.
- n number of points in the neighborhood p.
- q_i ranges over the neighborhood.
- I_{q_i} is the intensity of the pixel q_i.
- s is the shift.

The kernel function k:

- Should not introduce new extrema in f when smoothing.
- If it is a unit-area box function this reduces to standard histogram binning.
- Usually k is chosen as a Gaussian.

Locally Weighted Smooth Histogram

The smoothed locally weighted histogram is given by:

$$\hat{f}_p(s) = \sum_i K\left(I_{\boldsymbol{q_i}} - s\right) W\left(\boldsymbol{p} - \boldsymbol{q_i}\right) \tag{1}$$

- W is a weighting function which is:
  - Positive
  - Has unit-sum
  - Pixel influence drops off with distance from the $\boldsymbol{p}$

In 2D, equation (1) can be thought of as a spatial convolution:

$$\hat{f}_p(s) = K\left(I_{\boldsymbol{p}} - s\right) * W \tag{2}$$

- K determines the frequency content of $\hat{f}_p(s)$ as a function of s.
- W determines the spatial frequency content.
- For W arbitrary kernel, the convolution can performed at $O(\log(n))$ (for n neighborhood size) operations per output pixel using 2D FFT.
- If K, W are both Gaussian, the convolution can be done in constant time, independent of neighborhood size per pixel.

Histograms Properties - Modes

- The mode is the value that appears most often in a set of data.
- Number of modes within a neighborhood:
  - Single peak or mode - pixels in that neighborhood are members of the same population
  - Multiple modes - neighborhood contains pixels from two or more distinct populations.
- We would like to identify the number of modes, their value, widths, percentages of the population within each mode.
- For the smoothed histogram, a mode is defined by $\frac{\partial \hat{f}s}{\partial s} = 0$.

The smoothed local histogram as a convolution is given by (equation 2):

$$\hat{f}_p(s) = K(I_{\boldsymbol{p}} - s) * W$$

- W does not depend on s - the derivative of the histogram at pixel $\boldsymbol{p}$:

$$D_p(s) = \frac{\partial \hat{f} s}{\partial s} = -K^{'}(I_p - s) * W$$

- K is low pass filter, therefor its derivative K' is also band limited.
- We can sample $D_p(s)$ at or above Nyquist frequency of K' without loss of information.
- Defining $s_i, 1 \leq i \leq m$ a set of samples over the range of K', all histogram modes can be identified from the functions:

$$D_i(p) = -K^{'}(I_{\boldsymbol{p}} - s_i) * W \tag{3}$$

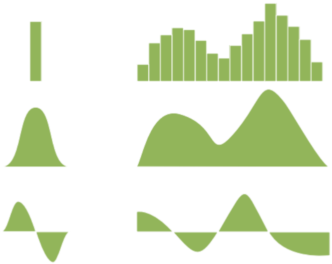
- The computation can be efficiently done by modern GPU hardware.
- Negative-going zero crossing in the function are the histogram modes.
- Positive-going zero crossing in the function are anti-modes.

Method for Finding Histogram Modes

- For each i, create a look up table L_i which maps any intensity value $I_p \mapsto K^{'}\left(s_i - I_p\right)$.
- The input image is mapped through the look up table.
- The results are convolved with the spatial kernel W to get the function D_i
- By increasing the sampling rate sufficiently, linear interpolation in s is accurate as desired.
- With sufficient sampling we can calculate the modes of $\hat{f}_p$:
  - At each point $\boldsymbol{p}$ we look for negative-going zero crossing in $D_i\left(\boldsymbol{p}\right)$
  - if a zero crossing if found between $D_i\left(\boldsymbol{p}\right)$ and $D_{i+1}\left(\boldsymbol{p}\right)$ there's a mode located at:

$$s = s_i + \frac{D_i\left(\boldsymbol{p}\right)}{D_i\left(\boldsymbol{p}\right) - D_{i+1}\left(\boldsymbol{p}\right)} \cdot \left(s_{i+1} - s_i\right)$$

Histograms Properties



Left: Look up table. Top Right: Raw histogram. Middle: Smoothed histogram. Bottom: Derivative of Smoothed Histogram

Closest Mode Filter

- Closest mode to be the mode one would reach by steepest ascent in the smoothed local histogram.
- Estimate $D(I_p)$, if the derivative is positive, use the rst mode greater than the pixel value, otherwise use the rst mode smaller than the pixel value.
- Greatly relies on the central pixel value.

Is it always the best choice?

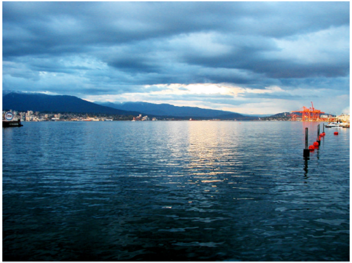
- In the presence of low variance noise, the mode closest to each may not be the best choice.

Mean Filter

- Robust to noise in the extrema.
- Does not use the central pixel value to choose the dominant mode.
- The filter is implemented by look up tables and convolution at constant time.

Erosion and Dilation Filters

- Modifying the erode and dilate operators to the 5% and 95% percentile modifies the results to the traditional operators.
- Robust against noise.
- In this case, unequal neighborhood weighing affects the results.



Dominant Mode Filter

- The median filter which uses 50% as a fixed point.
- It is possible to use a robust criterion to choose among the local modes.
- Using equation $s = s_i + \frac{D_i(\boldsymbol{p})}{D_i(\boldsymbol{p}) - D_{i+1}(\boldsymbol{p})} \cdot (s_{i+1} - s_i)$ we look for both negative and positive zero crossing - corresponding to modes and anti modes.
- Integrate between two anti-modes for each mode.
- Choose the mode with the largest integral between the two adjacent anti modes.
- The method produces sharper edges than the median for certain structures.

Mode Filters - Side By Side

(a) Original With Noise

(b) Closest Mode

(c) Bilateral

(d) Channel Smoothing

(e) Median

(f) Dominant Mode

Filters in Action - Detail Enhancement



Filters in Action - Detail Enhancement

(a) Original.

(b) After multi-layer contrast boost.

Summary

- Local image histogram are an important tool in visual computing - mean filter, erosion and dilation.
- Using smoothed histogram allows one to work with large neighborhoods in constant time, regardless of their size.
- The closest mode filter can be used to reduce noise in an image, but is not robust to low variance mode.
- Allows more robust implantation of the erosion and dilation filters.
- Dominant mode filter is both robust to low variance noise and allows edge sharpening.
- Other applications include detail layers extraction and detail enhancement.

- Kass, Michael, and Justin Solomon. '"Smoothed Local Histogram Filters." ACM Transactions on Graphics 29.4 (2010): 1. Print.
- Dorin Comaniciu and Peter Meer. "Mean shift: A robust approach toward feature space analysis." IEEE Trans. Pattern Anal. Machine Intell , 24 , 603-619, 2002.