Anisotropic Diffusion Using Power Watersheds
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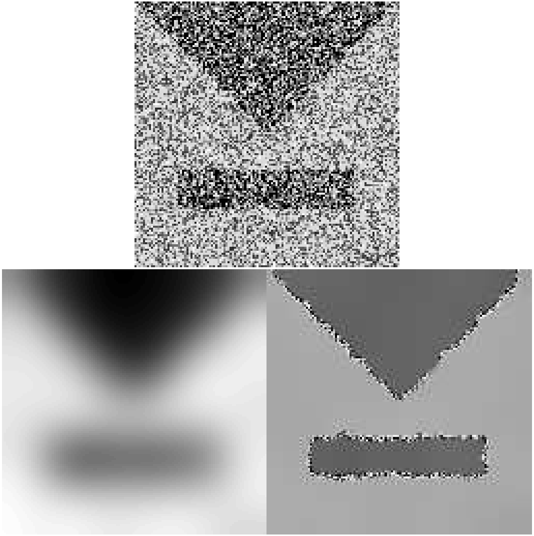
November 27, 2012

Motivation



Top: original image. **Down left**: Homogeneous Diffusion. **Down Right**: Perona Malik [3].

Motivation

- Image filtering using optimization.
- Blind diffusion filters are fast but blur edges.
- Anisotropic filters preserve edges but have extra cost.
- This method takles this trade-off proplem.

Outline

Introduction

- Anisotropic diffusion: gradient descent method for optimizing a robust error function by Black et al.[1]
- PW was introduced for image segmentation.
- PW good for addressing the robust estimator filtering model.
- Alternative to anisotropic diffusion.

Advantages of PW vs Anisotropic Diffusion

- No robust estimator parameter.
- Fast optimization while preserving discontinueties.
- No time Step.

Graph Notation

- $G = (V, E)$.
- e_{ij} is an edge between v_i and v_j.
- w_{ij} is the weight of e_{ij}.
- Edge-node incidence matrix A:

$$A_{e_{ij} v_k} = \begin{cases} +1 & \text{if the node } i = k \\ -1 & \text{if the node } j = k \\ 0 & \text{otherwise} \end{cases}$$

A is a combinatorial analogue of the continuous gradient operator

Anisotropic Diffusion Formulation

- $\frac{dx}{dt} = A^\top g(Ax)Ax$.
- x is the image intensities, $g(x)$ prevents blurring over edges.
- $g(x) = e^{-\alpha x}$, α a free parameter.
- Solved as:

$$x^{k+1} = x^k + dtA^\top g(Ax^k)Ax^k$$

Anisotropic Diffusion Formulation

- As the gradient of the energy $E(x) = \sigma(Ax)$, where $\sigma(x)$ a robust estimator
- $\frac{dE}{dx} = A^\top \sigma'(Ax)Ax$.
- As steady state optimization of the energy function

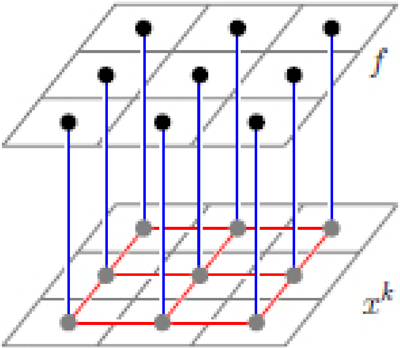
$$E_{k+1} = \sum_{e_{ij}} \sigma'(Ax^k)(x_i^{k+1} - x_j^{k+1})^2 + \lambda \sum_{v_i} \sigma'(x^k - f)(x^{k+1} - f)^2$$

- The generalized PW:

$$\min_x \sum_{e_{ij}} w_{ij}^p |x_i - x_j|^q + \sum_{v_i} w_i^p |x_i - y_i|^q$$

Initialization

The graph is built as in figure:



Compute the Weights

- Compute the pairwise weights: $e^{-(Ax^k)^2}$
- Compute the unary weights: $e^{-(x^k-f)^2}$

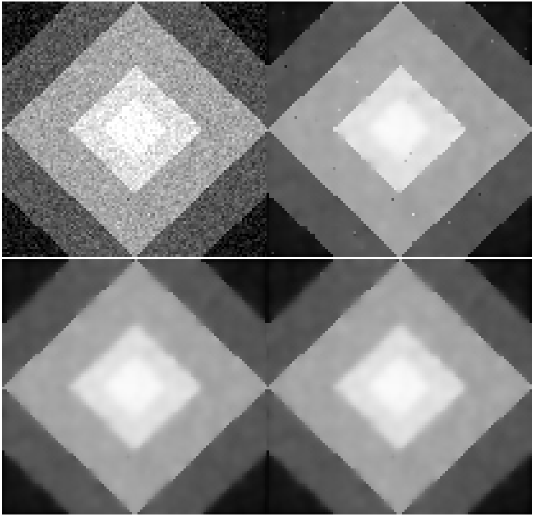
Algorithm

Initialize. Repeat:

- Compute the weights.
- Use PW with $y = f$ to optimize the energy function to obtain x^{k+1}.
- $k := k + 1$

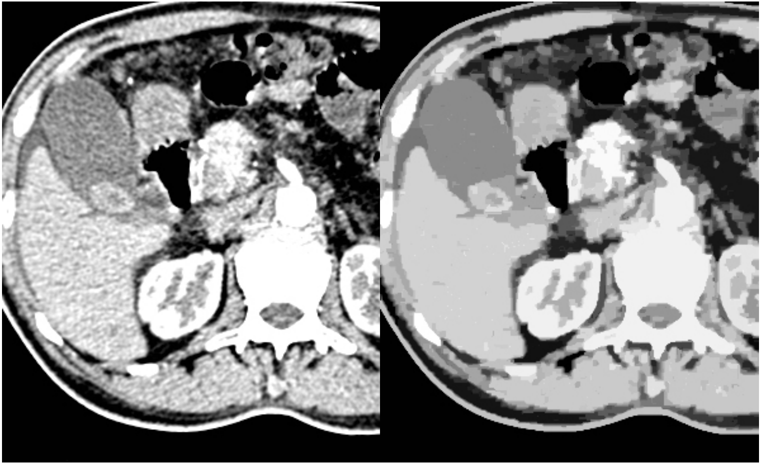
until $||x^{k+1} - x^k|| < \epsilon$

Results on Synthetic Images



Top left: Noisy image, PSNR = 24.24dB. **Top right**: PM, PSNR = 34.03dB, α = 0.0015. **Down left**: PM, PSNR = 30.46dB, α = 0.0005. **Down Right**: PW, PSNR = 31.54dB, λ = 0.975 [2].

Results on Real Images



left: Original image. **Down Right**: PW result [2].

Time Comparisons

		Perona-Malik		PW
Fig. 2, 104×100	Nb iter.	50	80	5
	Time (s)	0.19	0.30	0.17
Fig. 3, 299×364	Nb iter.	50	80	6
	Time (s)	1.95	3.08	2.43

Conclusion

- We used PW energy minimization for image filtering.
- PW optimization serves as a good denoising method on synthetic images.
- On real images, it is more to quantize the image into a small number of greyscales.
- It preserves edges but at the same time removes isolated noise.

References I

[1] M. J. Black, G. Sapiro, D. H. Marimont, and D. Heeger.
Robust anisotropic diffusion.
IEEE Trans. on Image Proc, 7:421–432, 1998.

[2] Camille Couprie, Leo Grady, Laurent Najman, and Hugues Talbot.
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IEEE International Conference on Image Processing, pages 4153–4156, 2010.

[3] Joachim Weickert.
Anisotropic Diffusion in Image Processing.
Teubner, Stuttgart, 1998.

THANK YOU

Questions?