Material for Chapter 6:

Basic Principles of Tomography

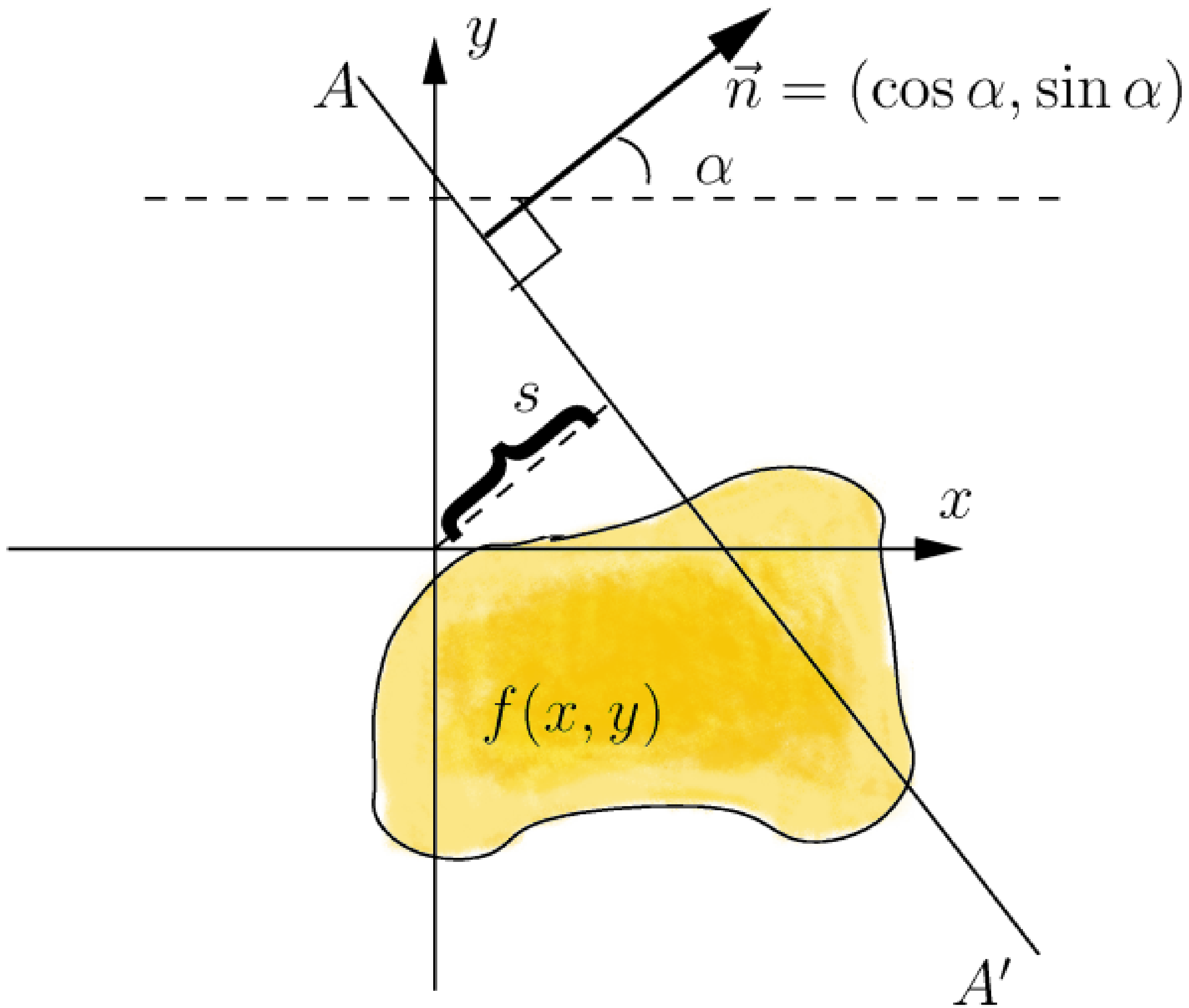


Figure 1: Radon Transform: Attenuation
Source: `http://en.wikimedia.org/wiki/Image:Radon_transform.png`

Parallel scanning geometry I

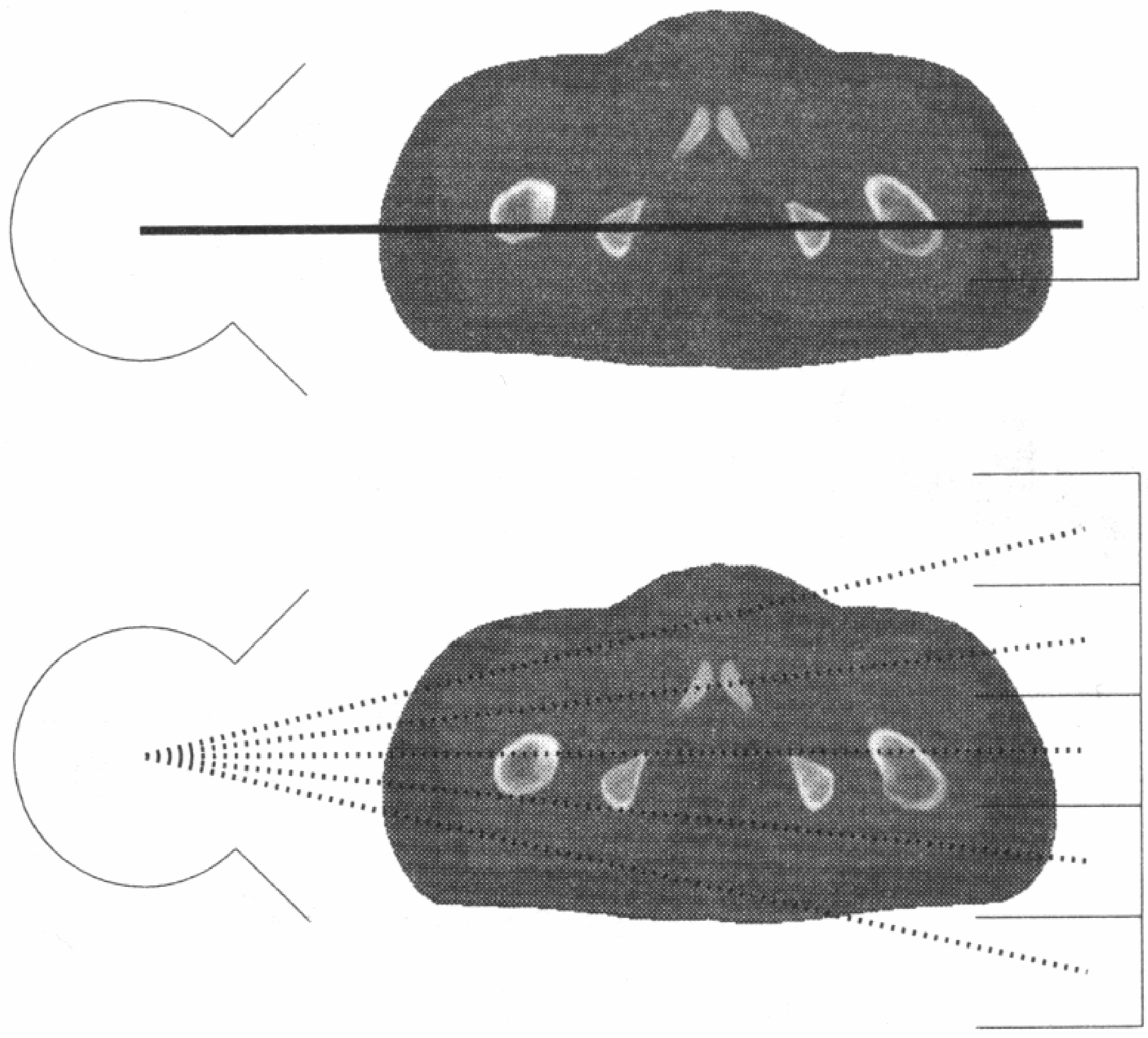


Figure 2: Dual motion scanner. **Top:** One source, one detector. **Bottom:** One source, several detectors. Source: Natterer & Wübbeling, 2001.

Parallel scanning geometry II

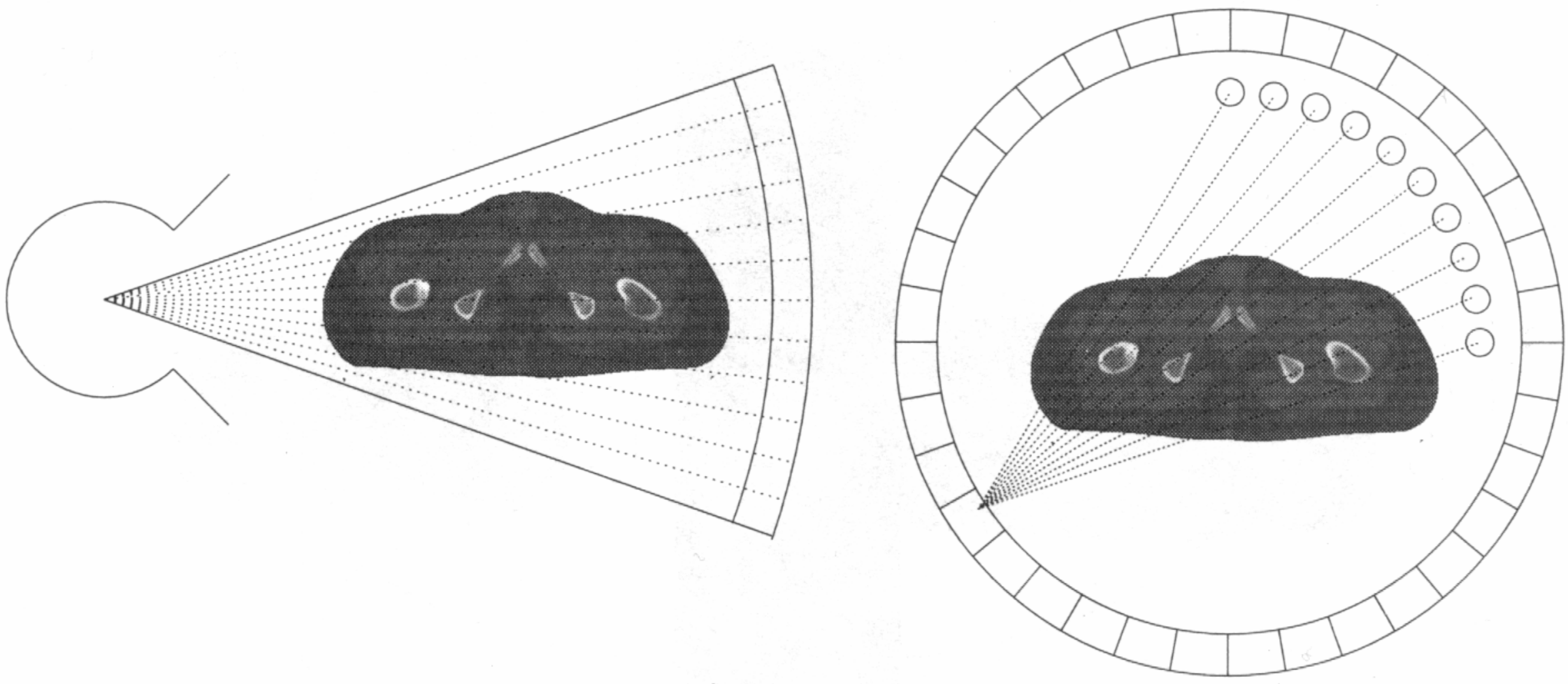


Figure 3: Fan beam scannng. **Left:** Rotating detector-source system. **Right:** Stationary detector ring, rotating source. Source: Natterer & Wübbeling, 2001.

Projections I

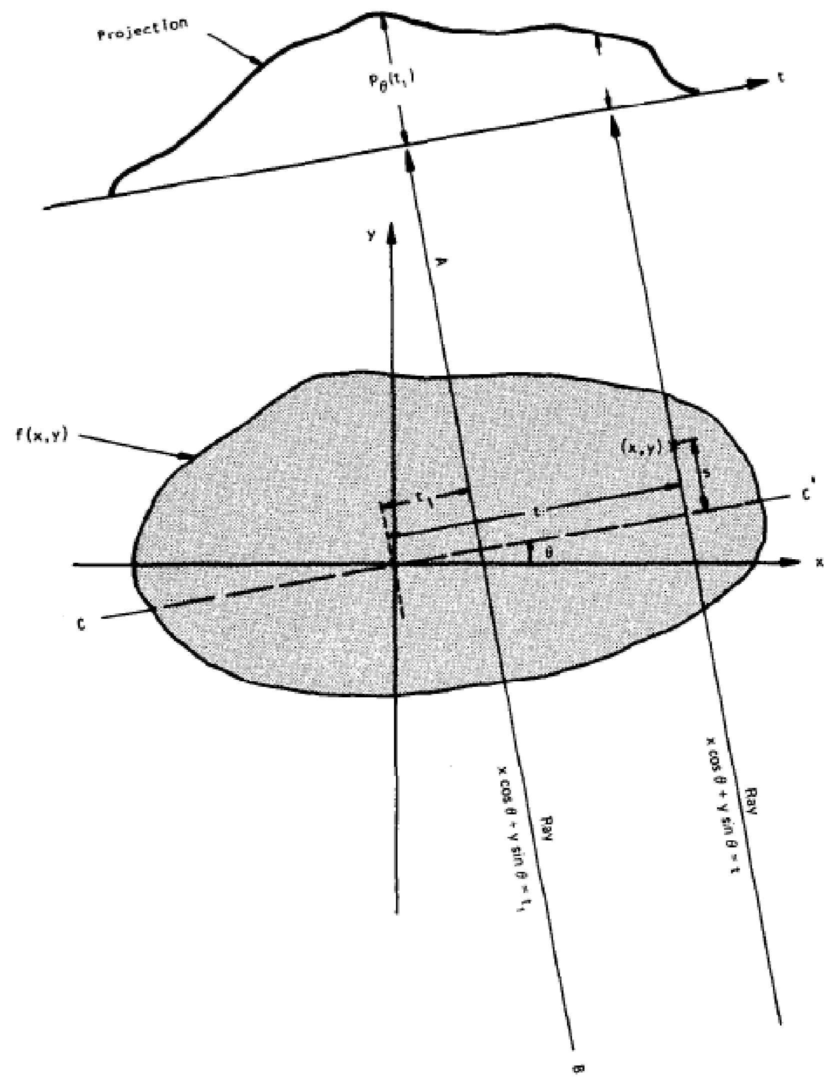


Figure 4: Object, $f(x, y)$, and its projection, $P_\theta(t_1)$, shown for angle θ. Source: Kak, 1979.

Parallel Projections

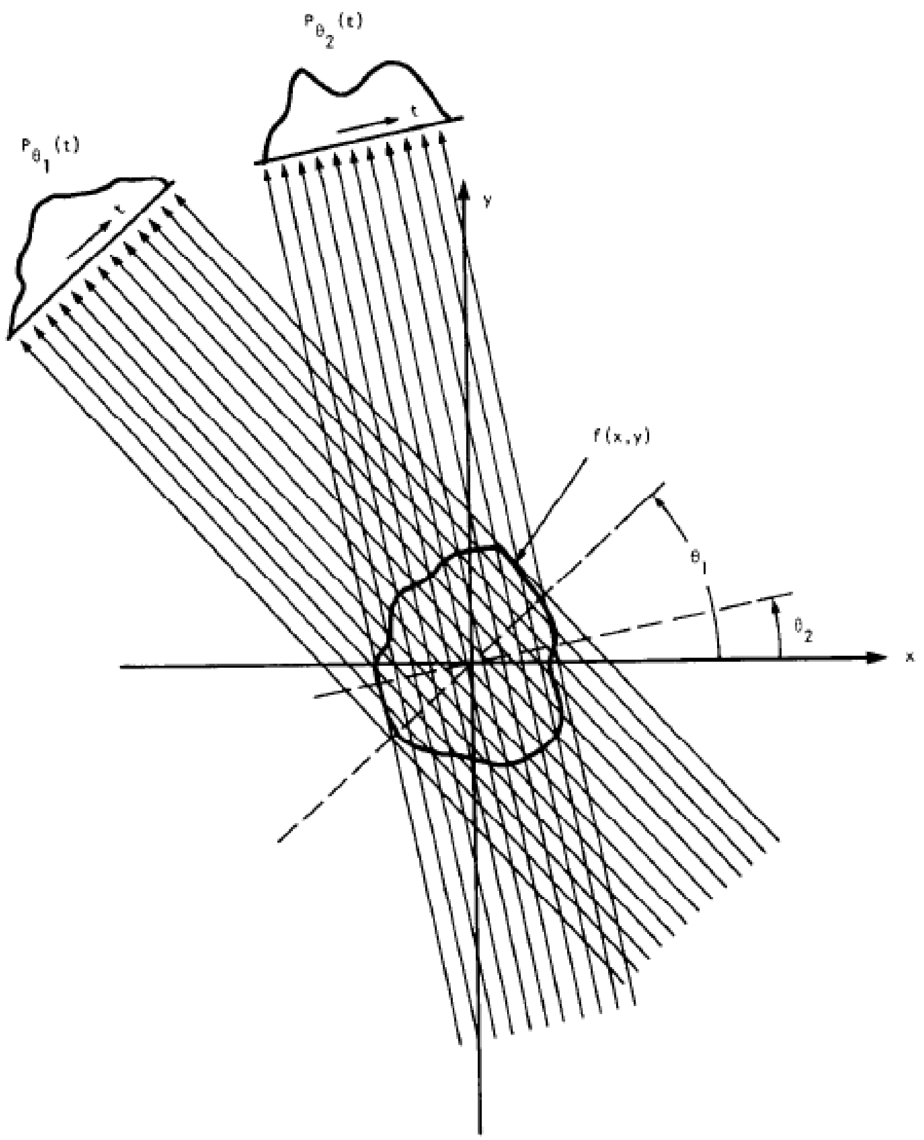


Figure 5: Parallel projections are taken by measuring a set of parallel rays for a number of different angles. Source: Rosenfeld & Kak, 1982.

Fan Beam Projections

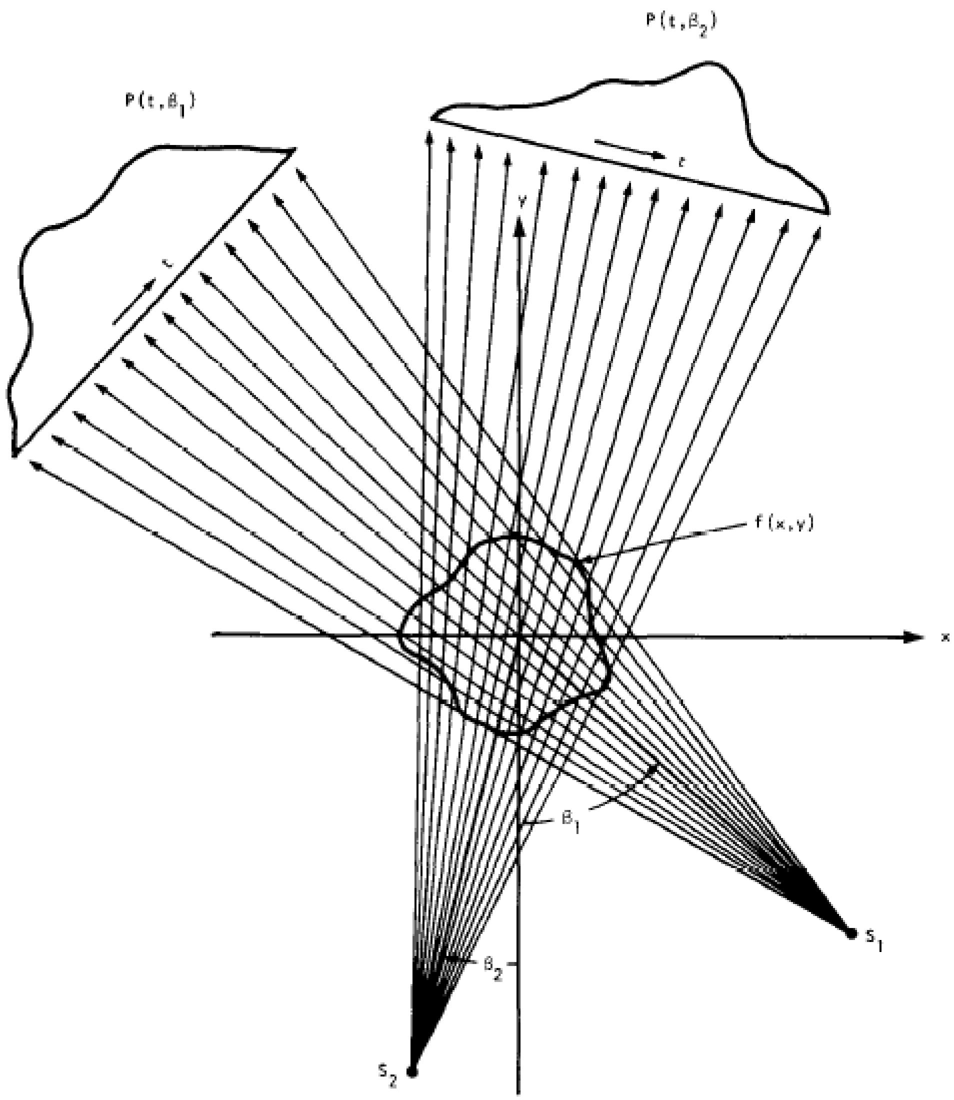


Figure 6: A fan beam projection is colleted if all the rays meet in one location. Source: Rosenfeld & Kak, 1982.

For tests: head phantom

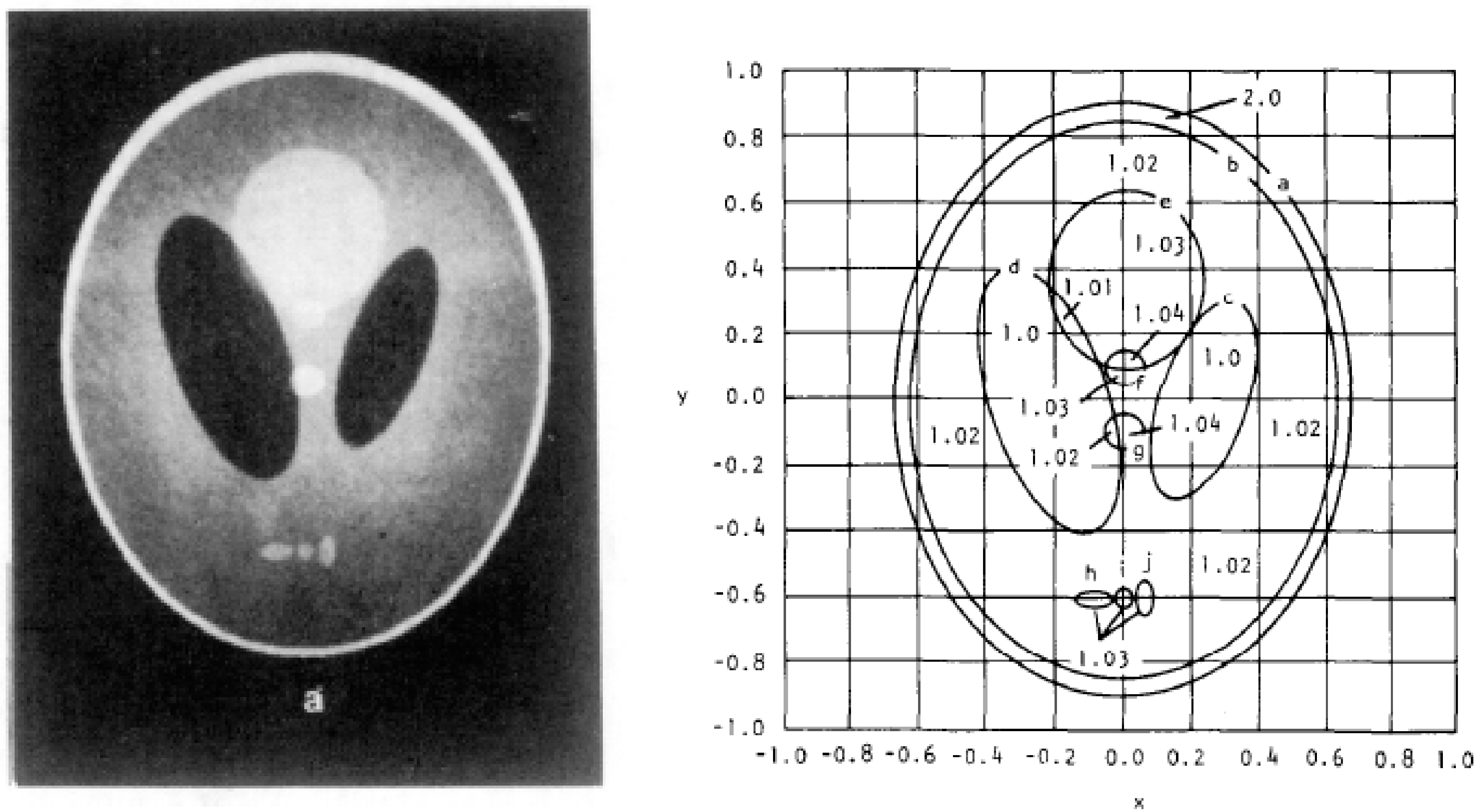


Figure 7: **Left:** The Shepp and Logan head phantom. **Right:** The phantom is a superposition of 10 ellipses. Source: Rosenfeld & Kak, 1982.

Example: Projected Ellipses I

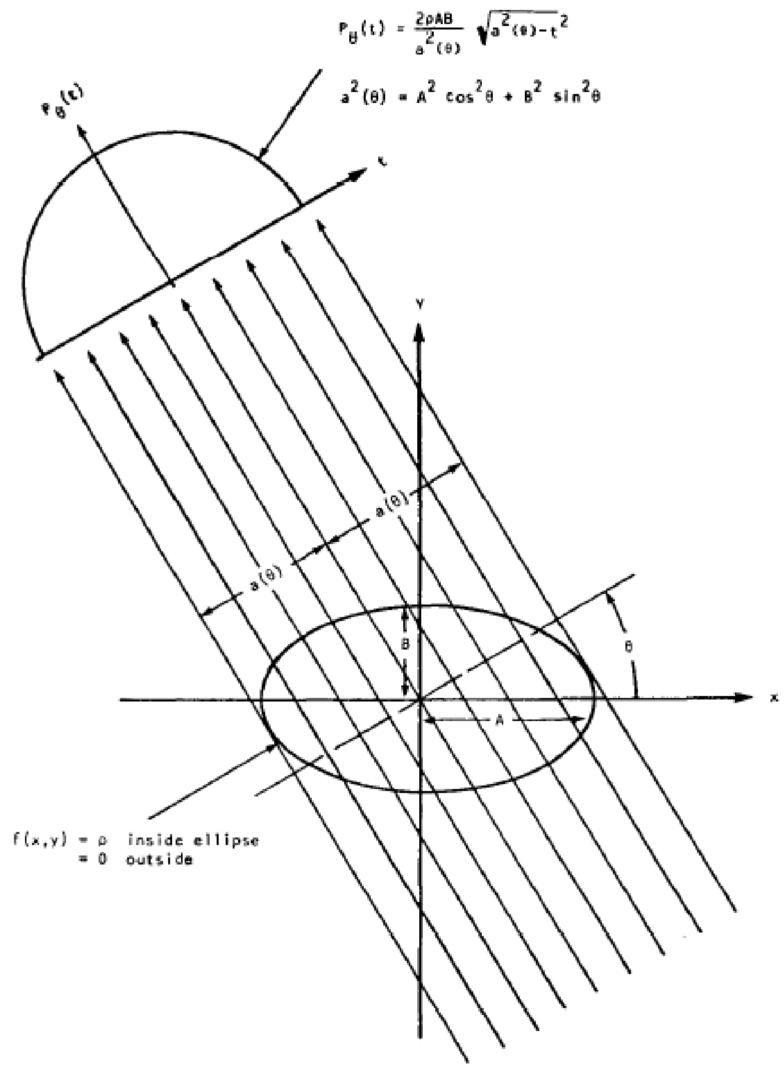


Figure 8: Analytic expression is shown for the projection of an ellipse. For computer simulations a projection can be generated by simply summing the projection of each individual ellipse. Source: Rosenfeld & Kak, 1982.

Example: Projected Ellipses II

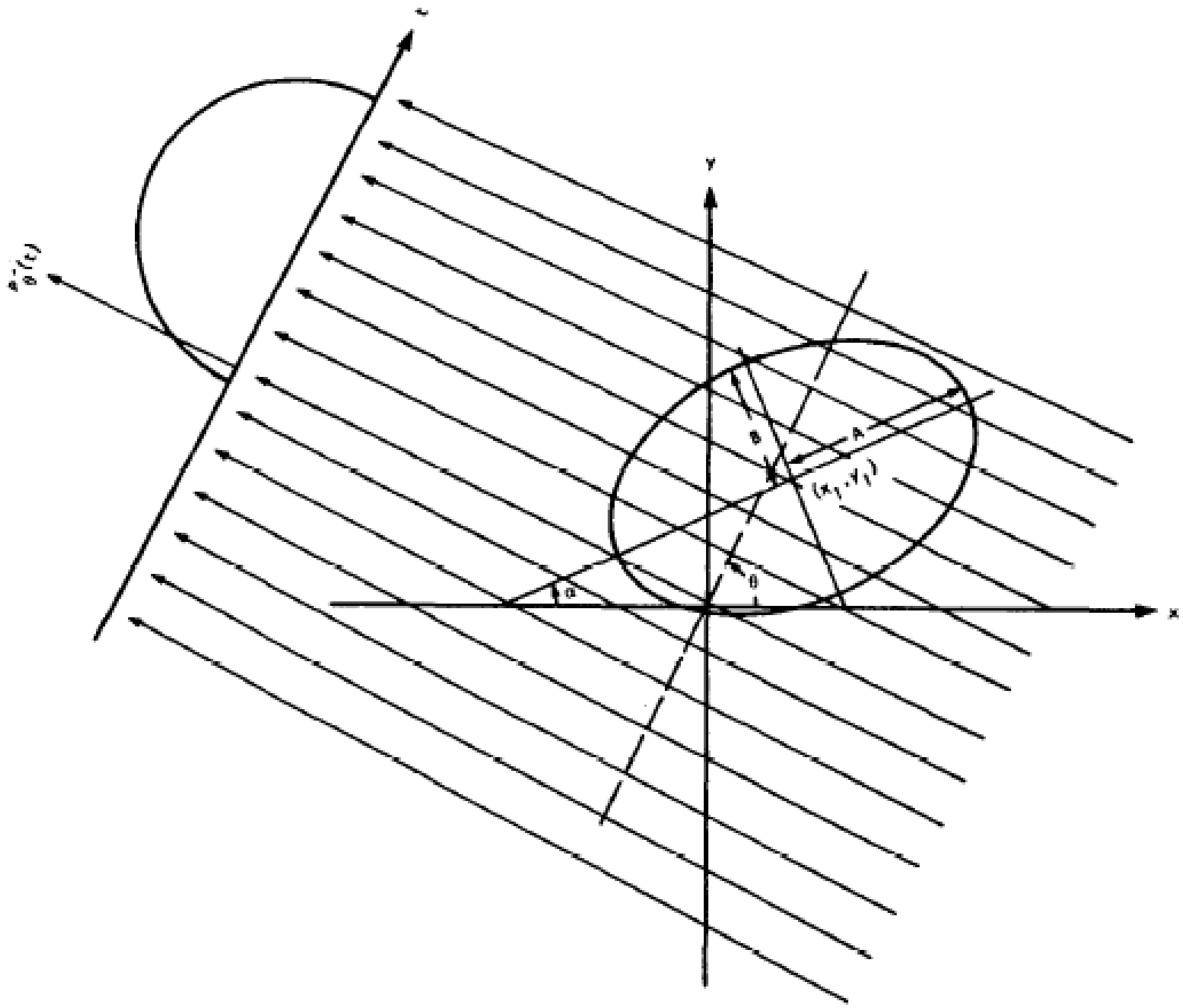


Figure 9: Analytical calculation for an ellipse with its center located at (x_1, y_1) and its major axis rotated by α. Source: Rosenfeld & Kak, 1982.

Fourier Slice Theorem

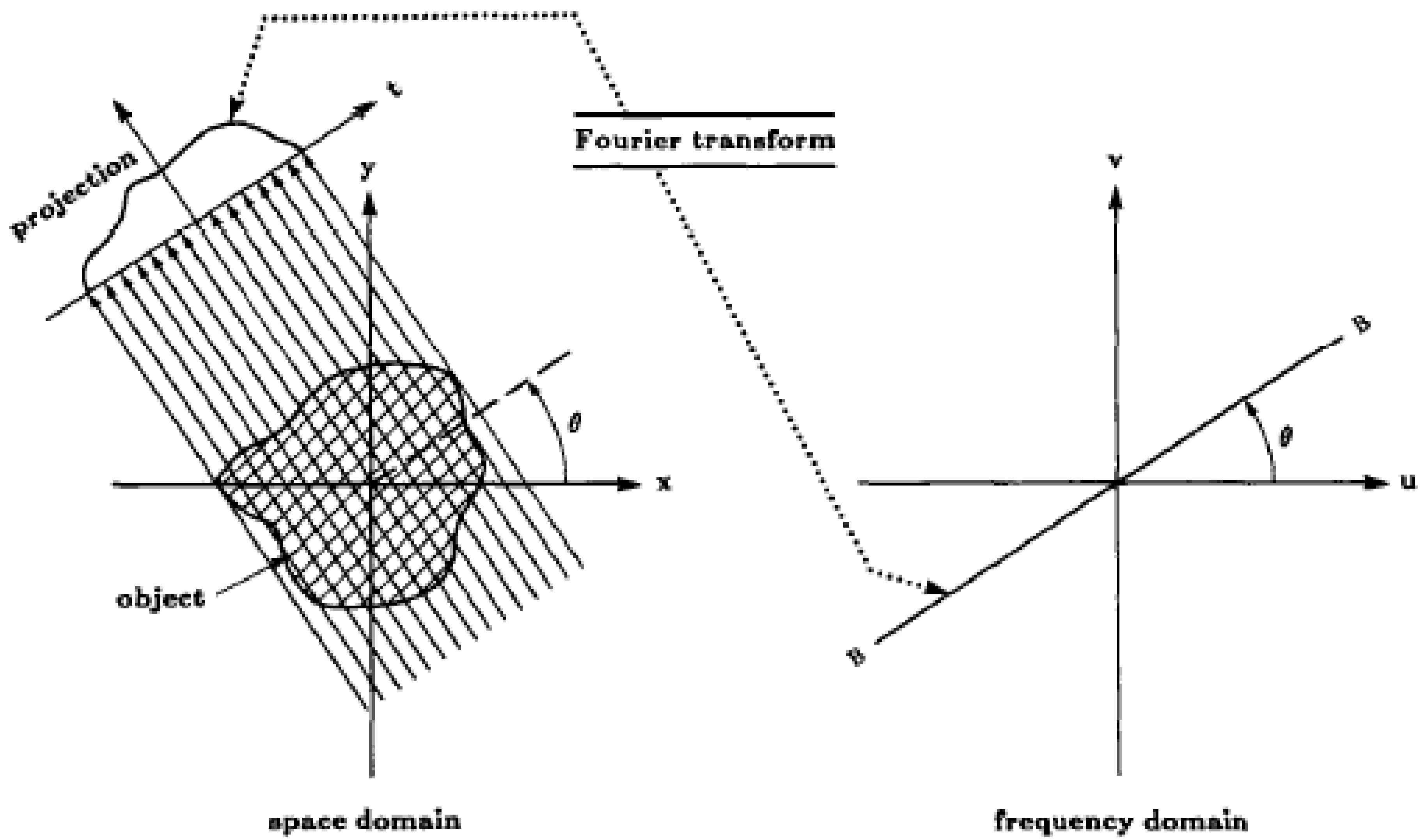


Figure 10: The Fourier Slice Theorem relates the Fourier transform of a projection to the Fourier transform of the object along a radial line. Source: Pan & Kak, 1983.

Interpolation in frequency domain ???

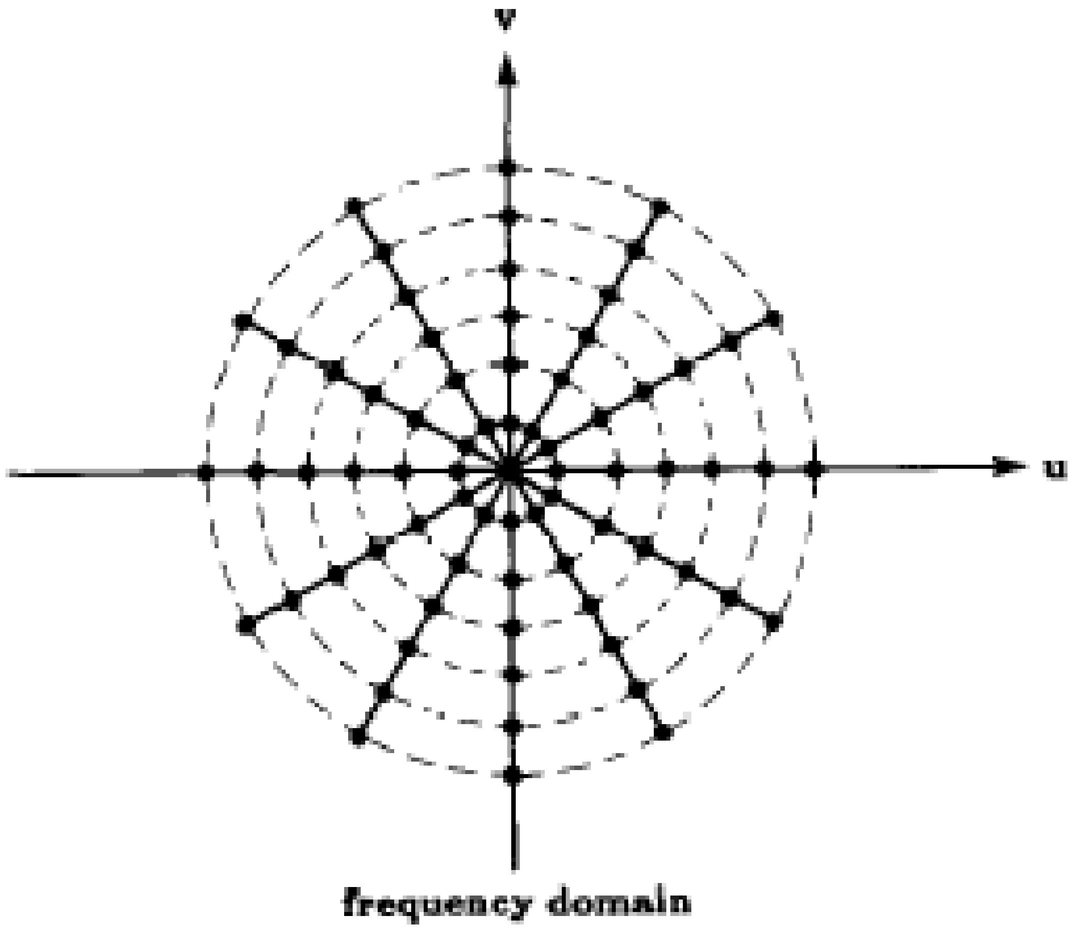


Figure 11: Collecting projections of the object at a number of angles gives estimates of the Fourier transform of the object along radial lines. The dots represent the actual location of estimates of the object's Fourier transform. Source: Pan & Kak, 1983.

Backprojection I

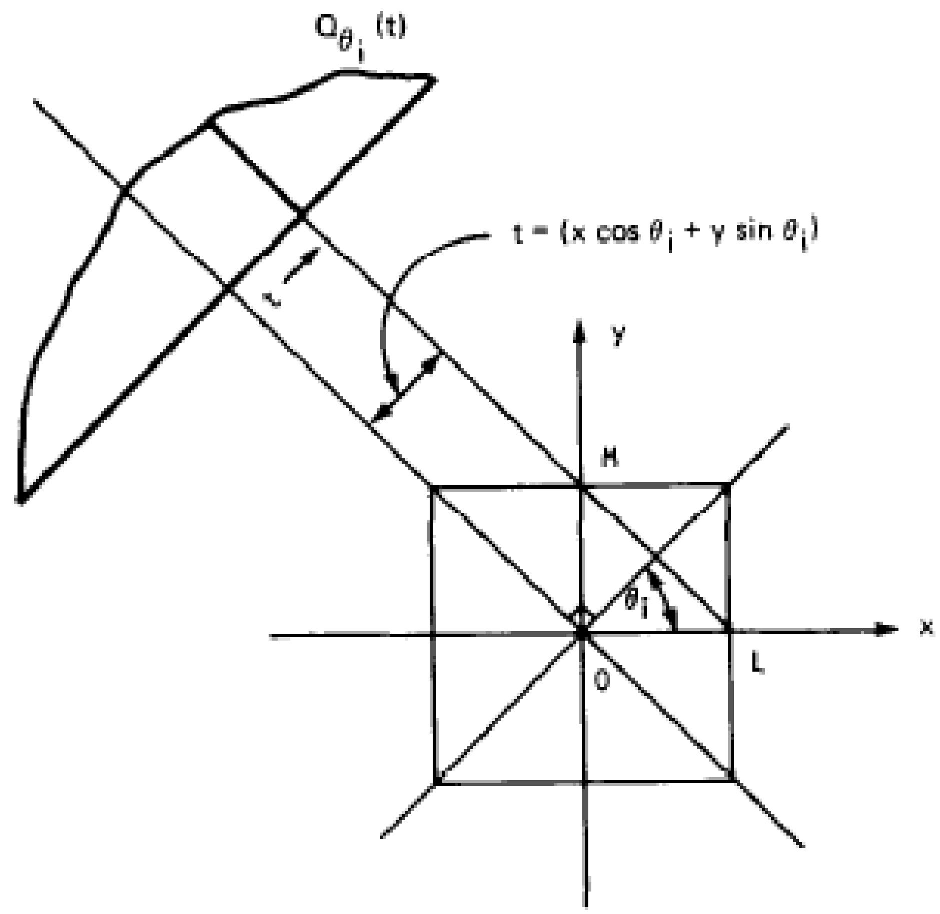


Figure 12: Reconstructions are often done using a procedure known as backprojection. Here a filtered projection is smeared back over the reconstruction plane along lines of constant t. The filtered projection at a point t makes the same contribution to all pixels along the line LM in the $x - y$ plane. Source: Rosenfeld & Kak, 1982.

Backprojection II

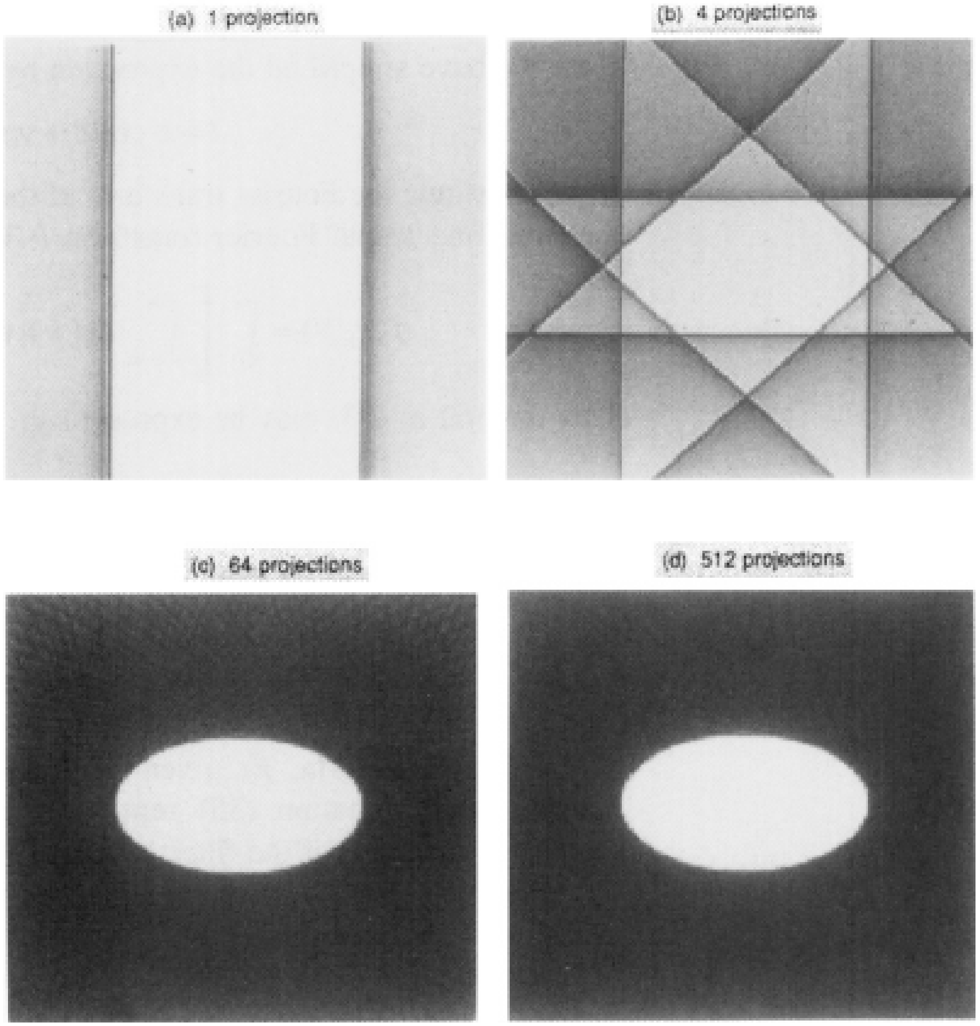
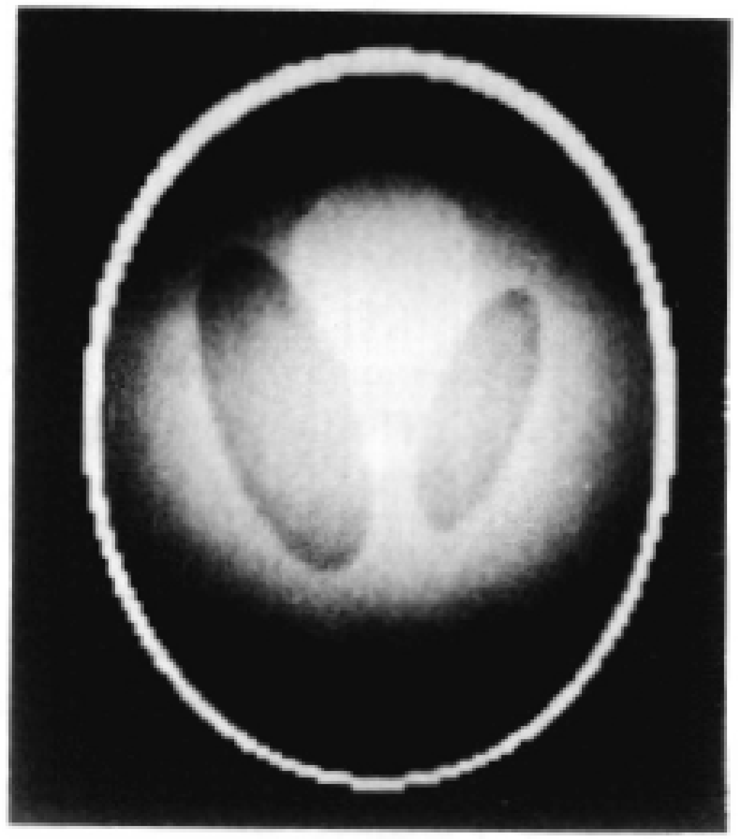


Figure 13: The result of backprojecting the projection of an ellipse. **(a)** shows the result of backprojecting for a single angle, **(b)** shows the effect of backprojecting over 4 angles, **(c)** shows 64 angles, and **(d)** shows 512 angles.

Example: Reconstruction

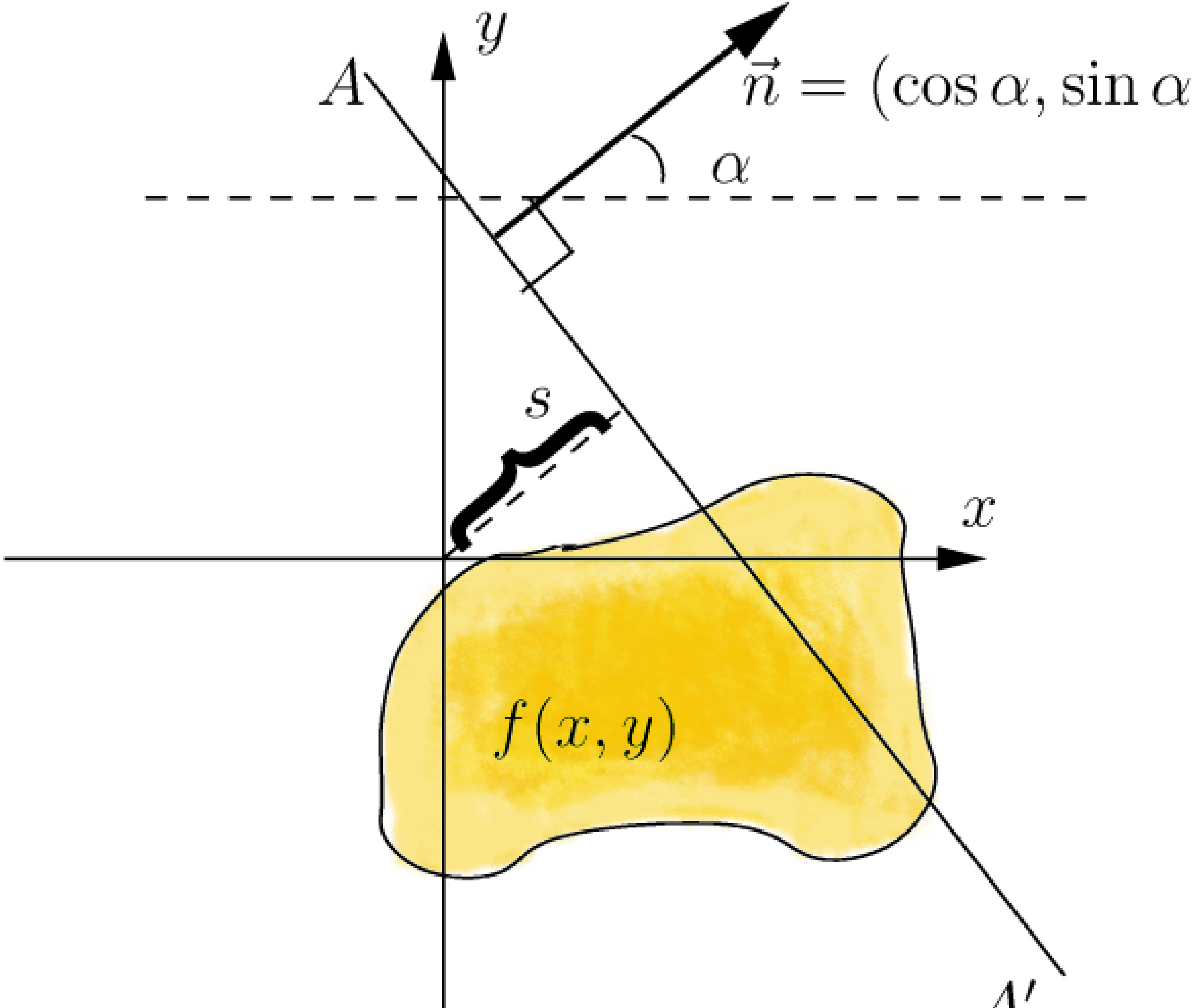


(a)

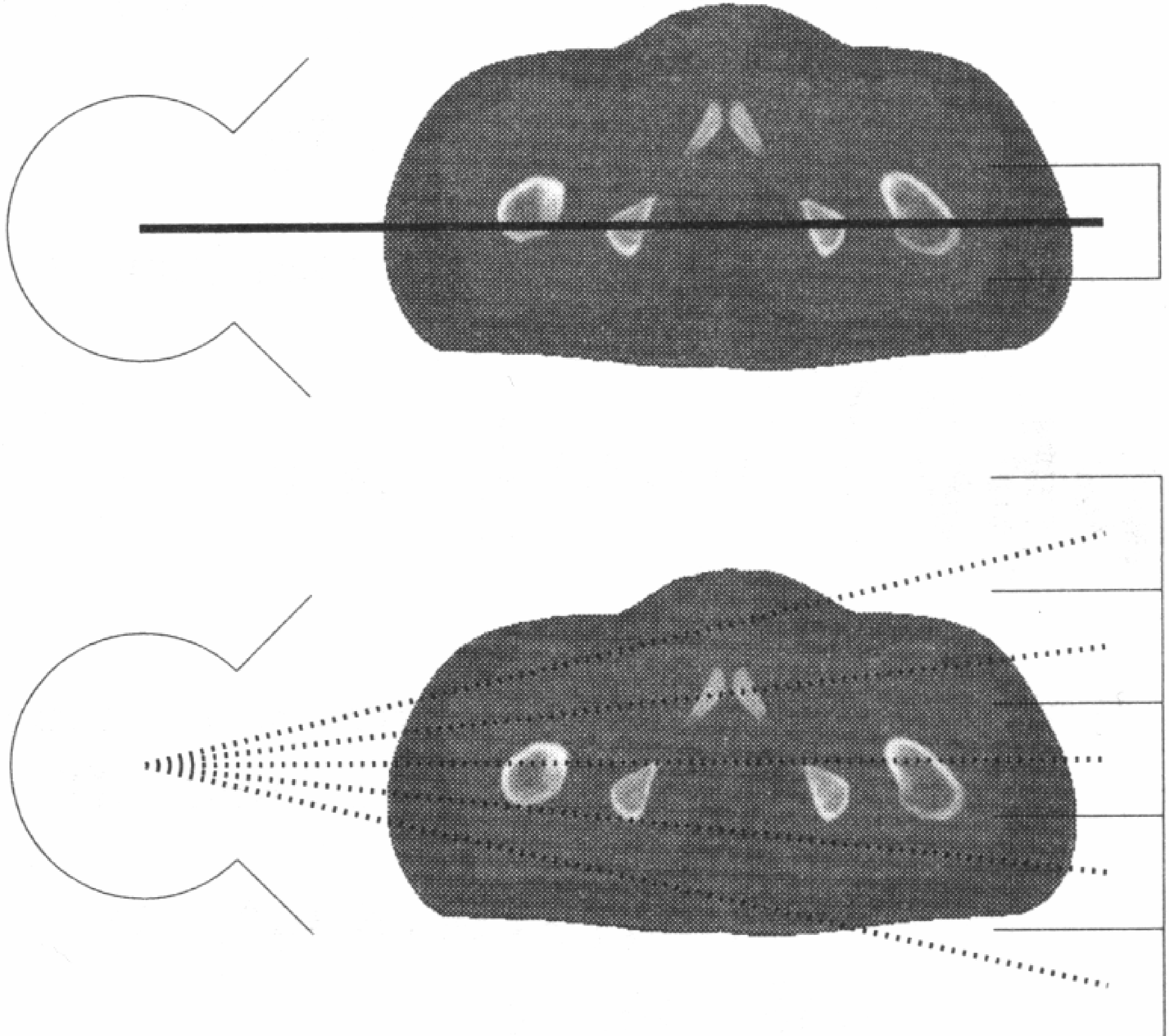
Figure 14: Reconstruction of the head phantom. The dark regions at the top and at the bottom are visible artefacts. This 128×128 reconstruction was made from 110 projections with 127 rays in each direction. Source: Rosenfeld & Kak, 1982

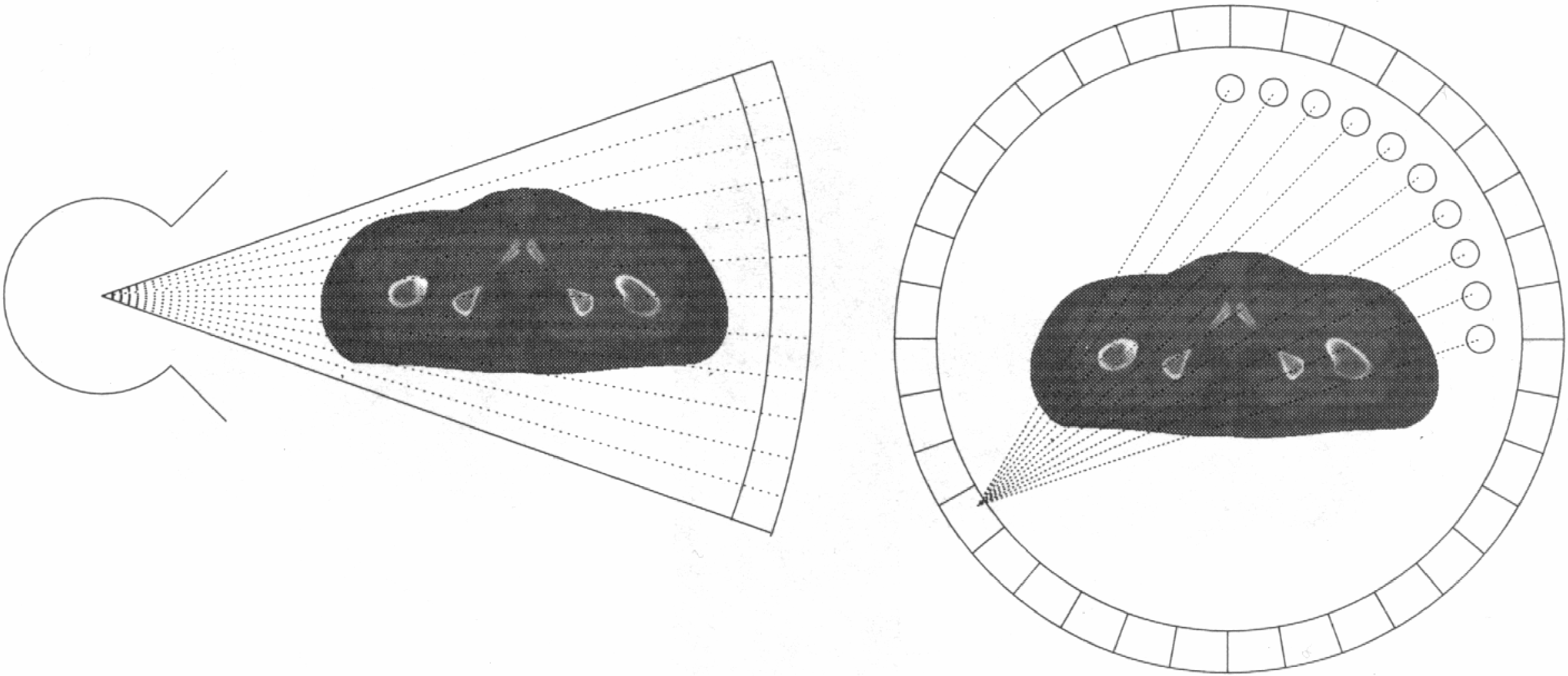
Literature

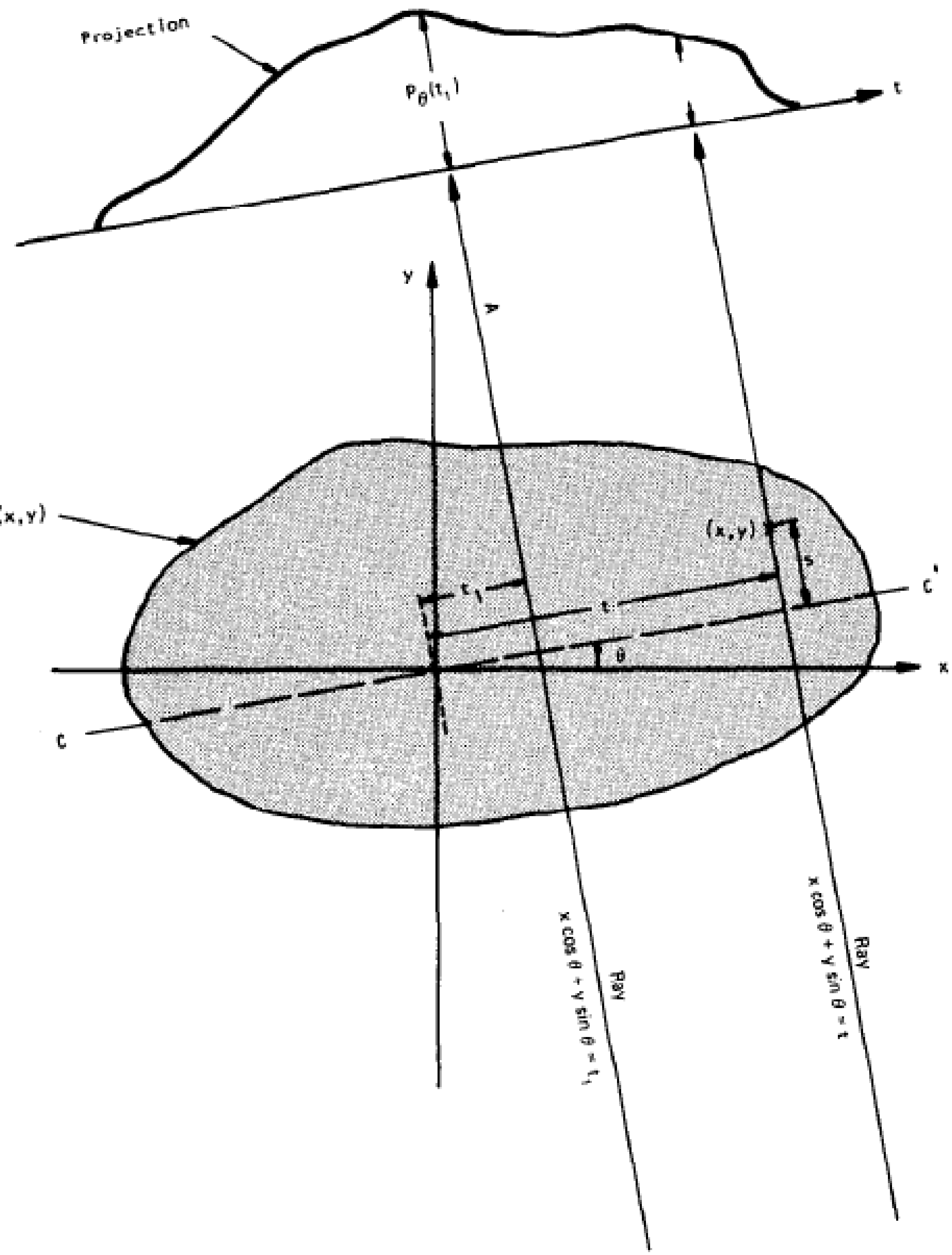
- F. Natterer, F. Wübbeling: *Mathematical Methods in Image Processing*. SIAM monographs on mathematical modeling and computation, 2001.
  (textbook with mathematical in-depth treatment of tomography)
- A.C. Kak:*Computerized tomography with X-ray emission and ultrasound sources*. Proc. IEEE, vol. 67, pp. 1245-1272, 1979.
- A. Rosenfeld and A.C. Kak: *Digital Picture Processing*. 2nd edition, Academic Press, New York, 1982.
- S.X. Pan and A.C. Kak: *A computational study of reconstruction algorithms for diffraction tomography: Interpolation vs. filtered- backpropagation*. IEEE Trans. Acoustic Speech Signal Processing, vol. ASSP-31, pp. 1262-1275, 1983.

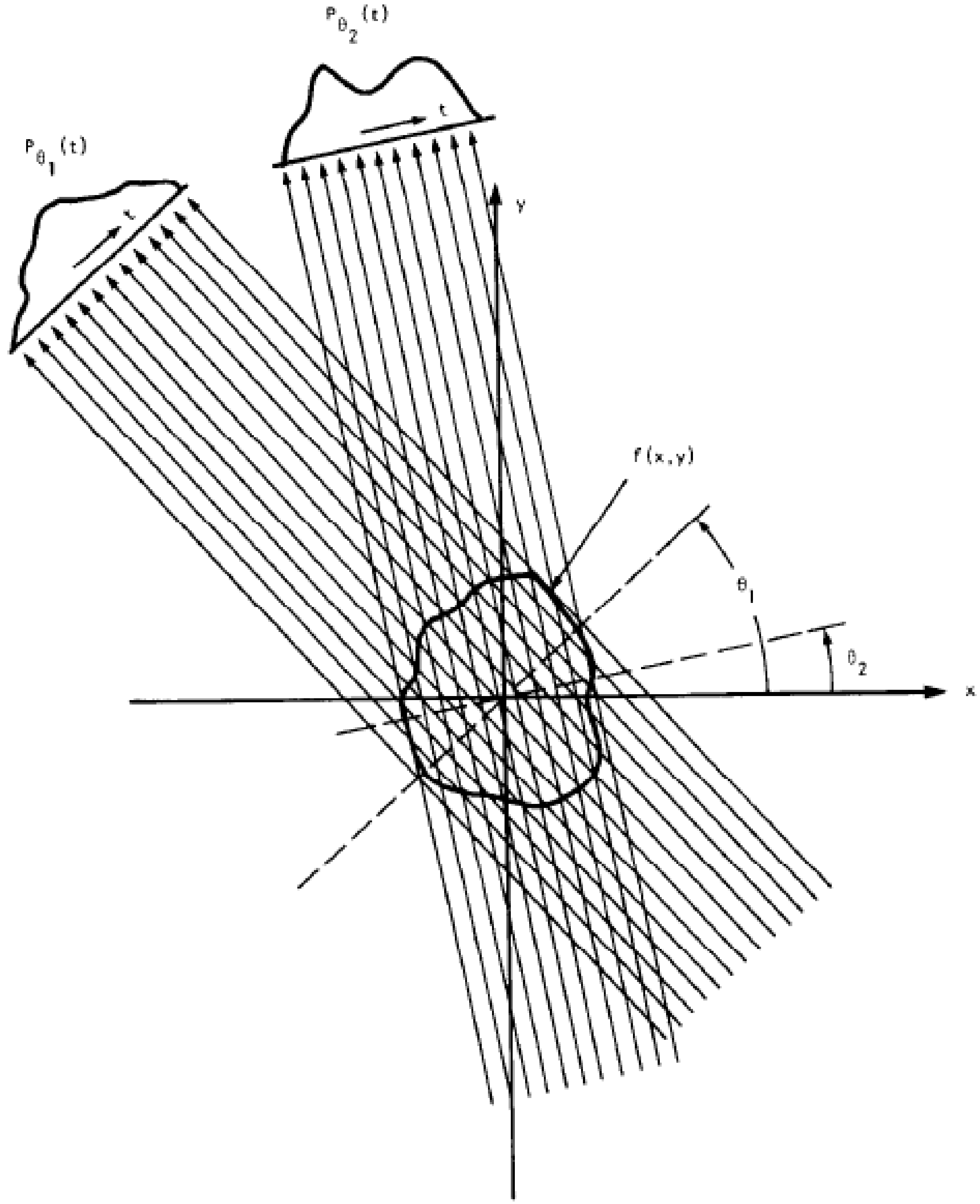


y
A
$\vec{n} = (\cos\alpha, \sin\alpha)$
α
s
x
$f(x, y)$
A′

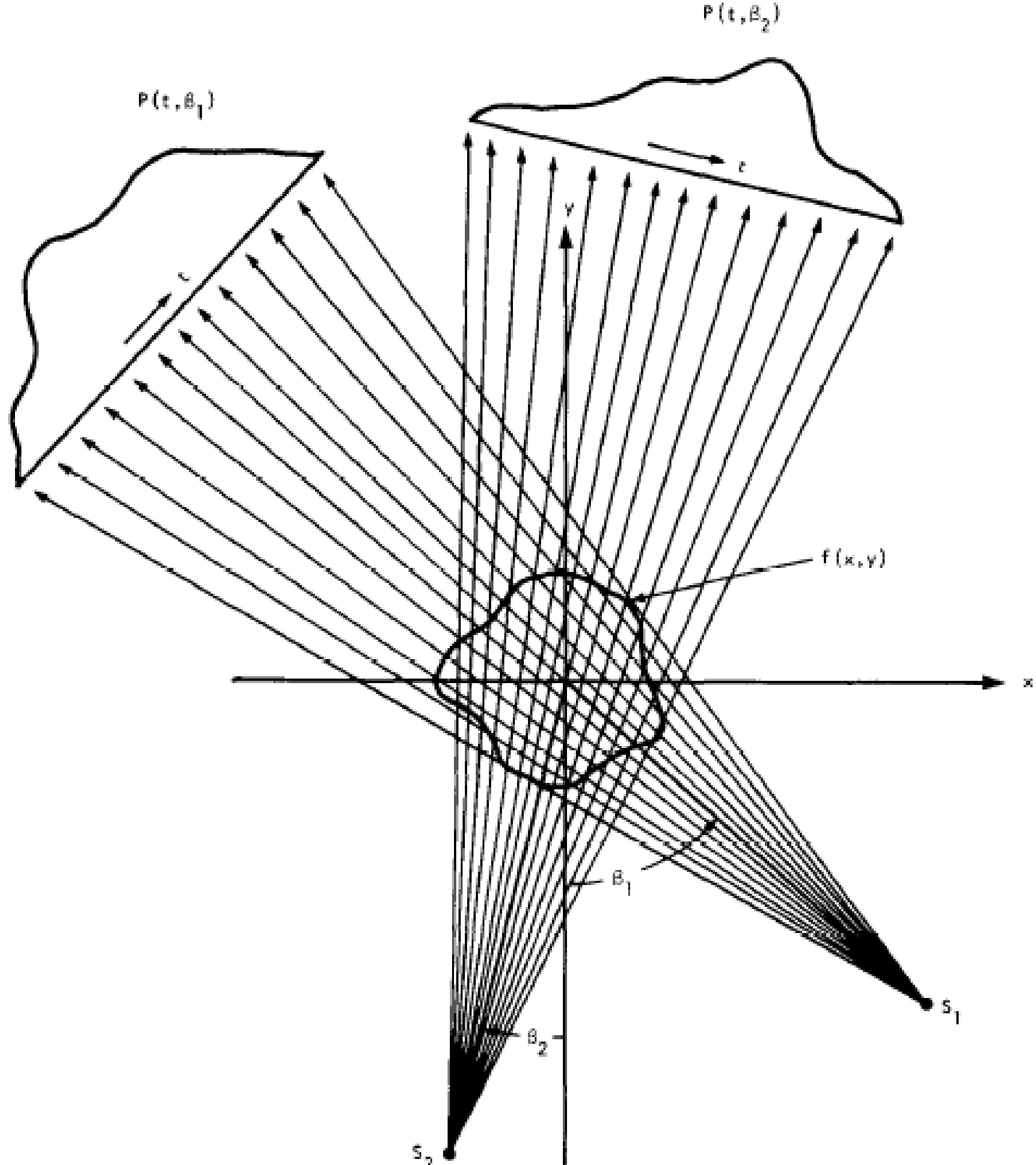




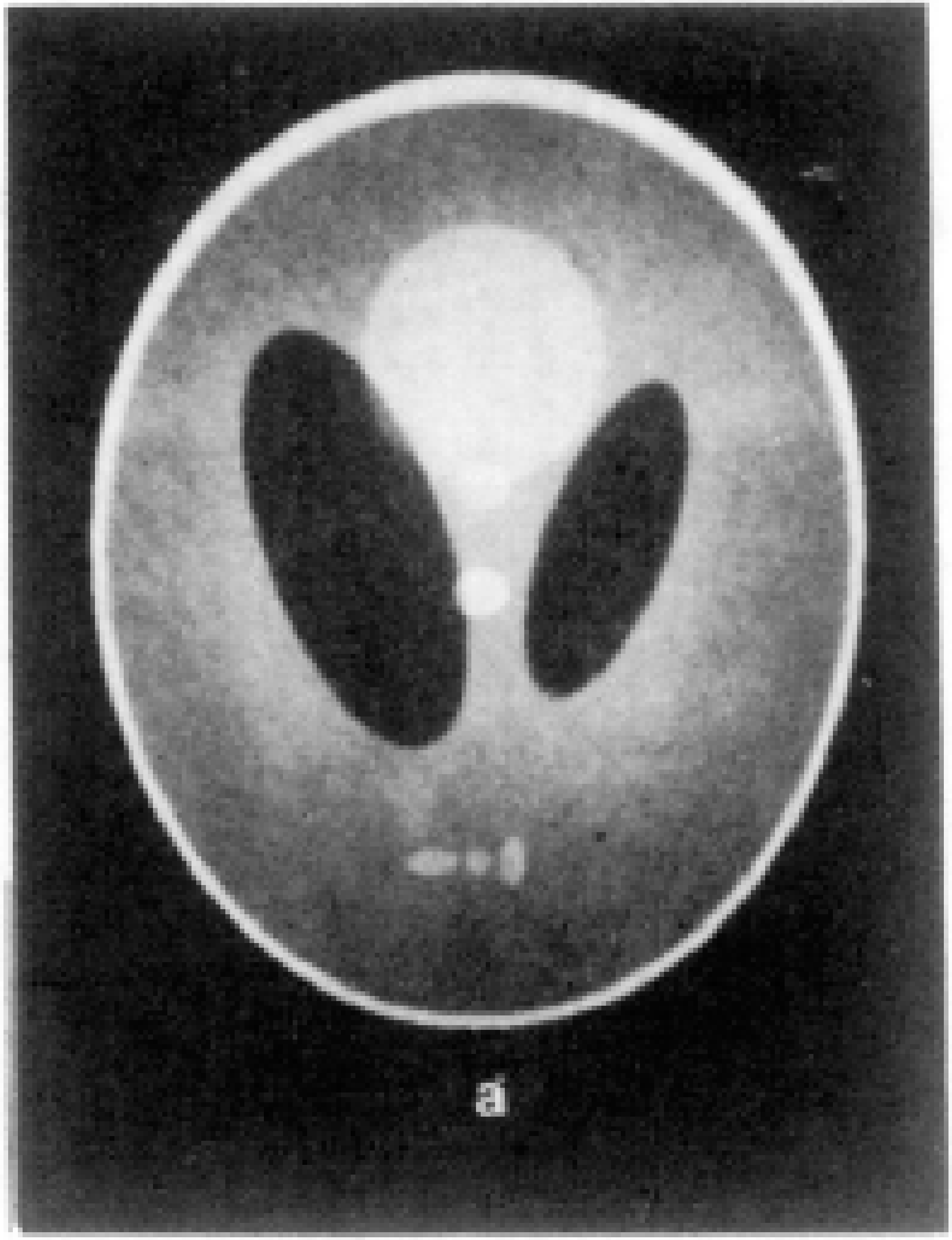

Projection
$P_\theta(t_1)$
t
y
A
f(x,y)
(x,y)
s
t_1
t
C'
θ
x
C
Ray
x cos θ + y sin θ = t_1
Ray
x cos θ + y sin θ = t
B

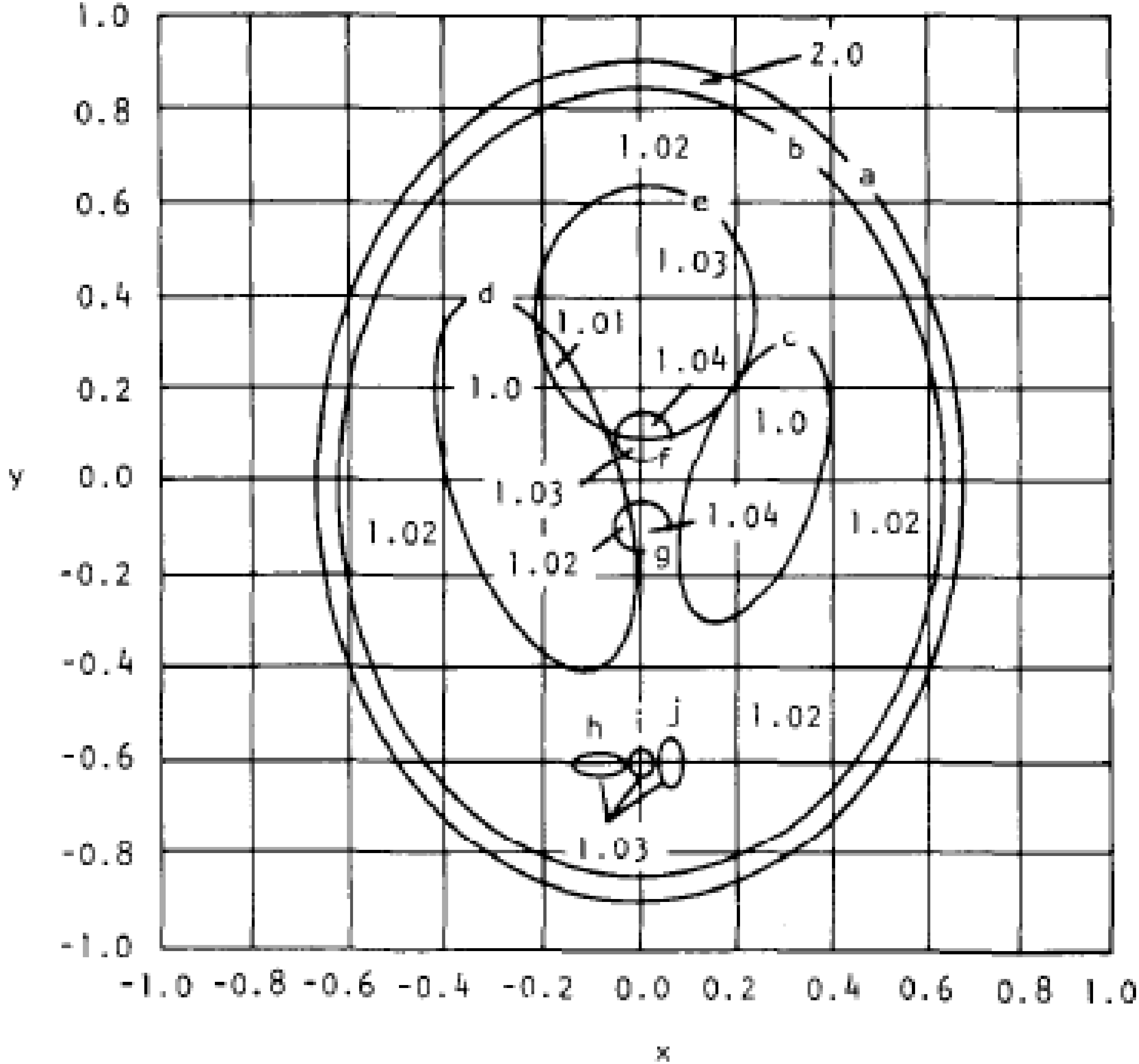


$P_{\theta_2}(t)$
$P_{\theta_1}(t)$
t
t
y
f(x,y)
θ_1
θ_2
x

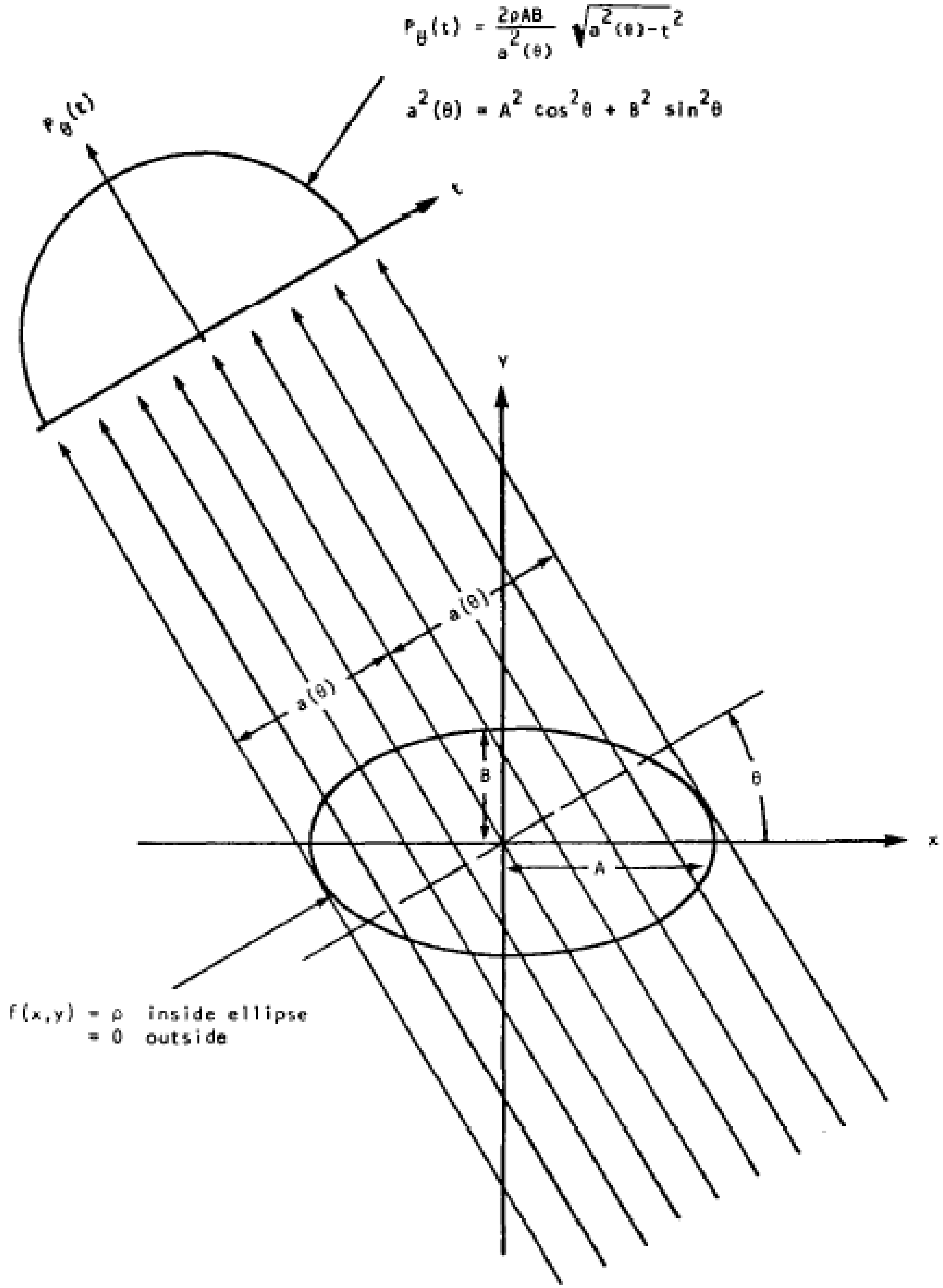


P(t, β2)
P(t, β1)
t
t
y
f(x, y)
x
β1
β2
S1
S2

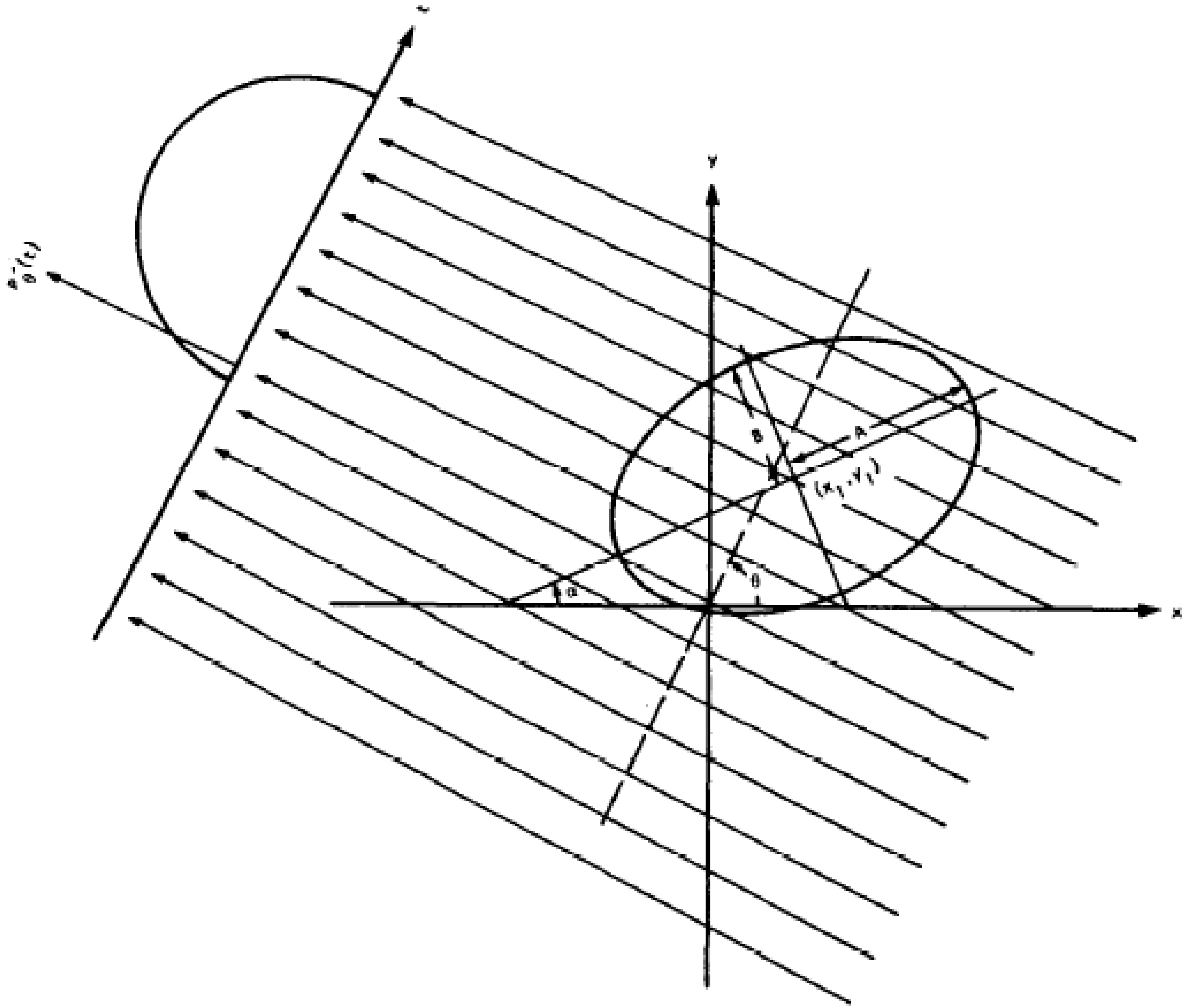

a



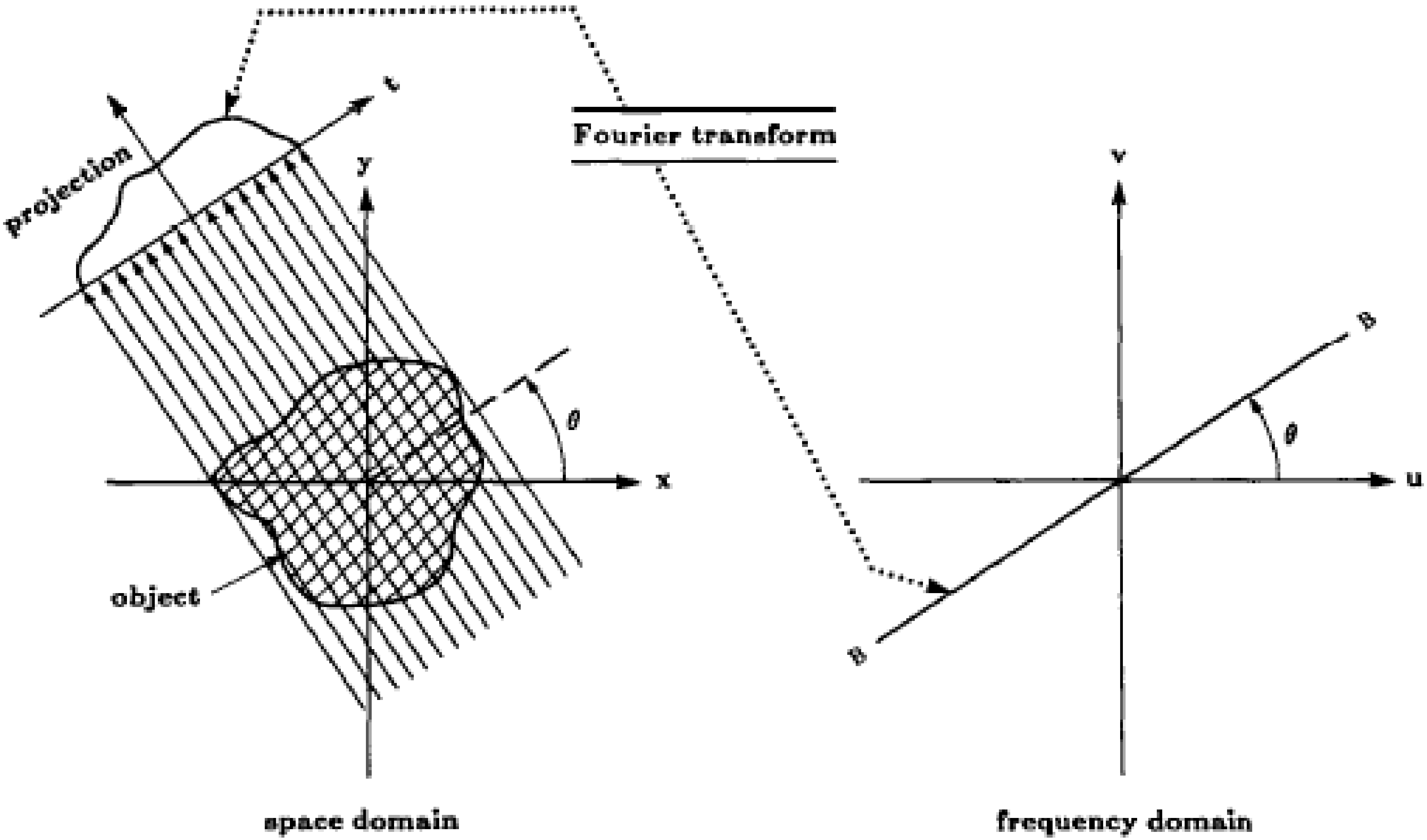
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
y
-1.0
-0.8
-0.6
-0.4
-0.2
0.0
0.2
0.4
0.6
0.8
1.0
x
2.0
a
b
c
d
e
f
g
h
i
j
1.02
1.03
1.01
1.04
1.0
1.0
1.03
1.04
1.02
1.02
1.02
1.02
1.03



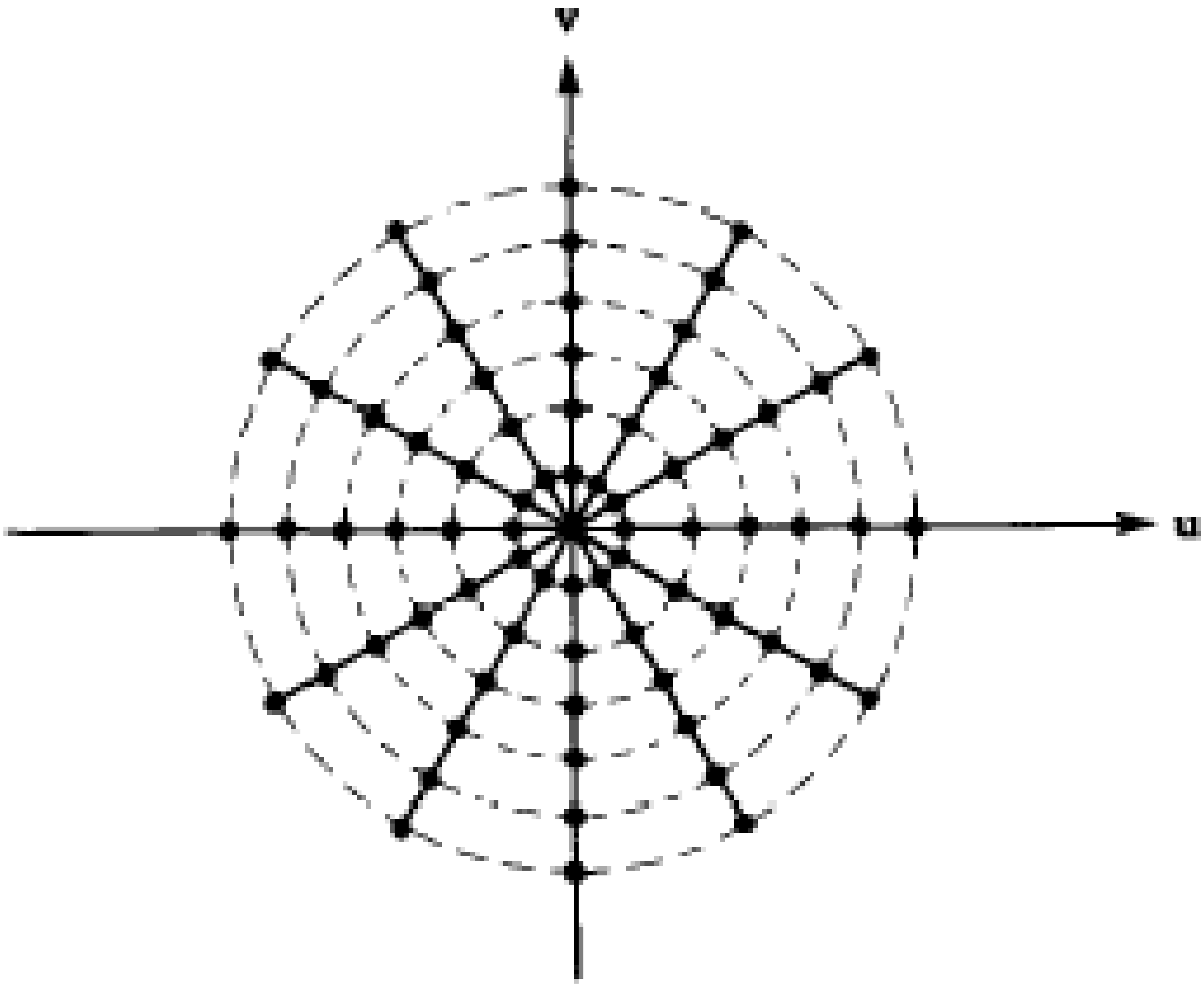
$P_\theta(t) = \frac{2\rho AB}{a^2(\theta)}\sqrt{a^2(\theta)-t^2}$
$a^2(\theta) = A^2\cos^2\theta + B^2\sin^2\theta$
$P_\theta(t)$
t
y
x
a(θ)
a(θ)
B
A
θ
f(x,y) = ρ inside ellipse
= 0 outside



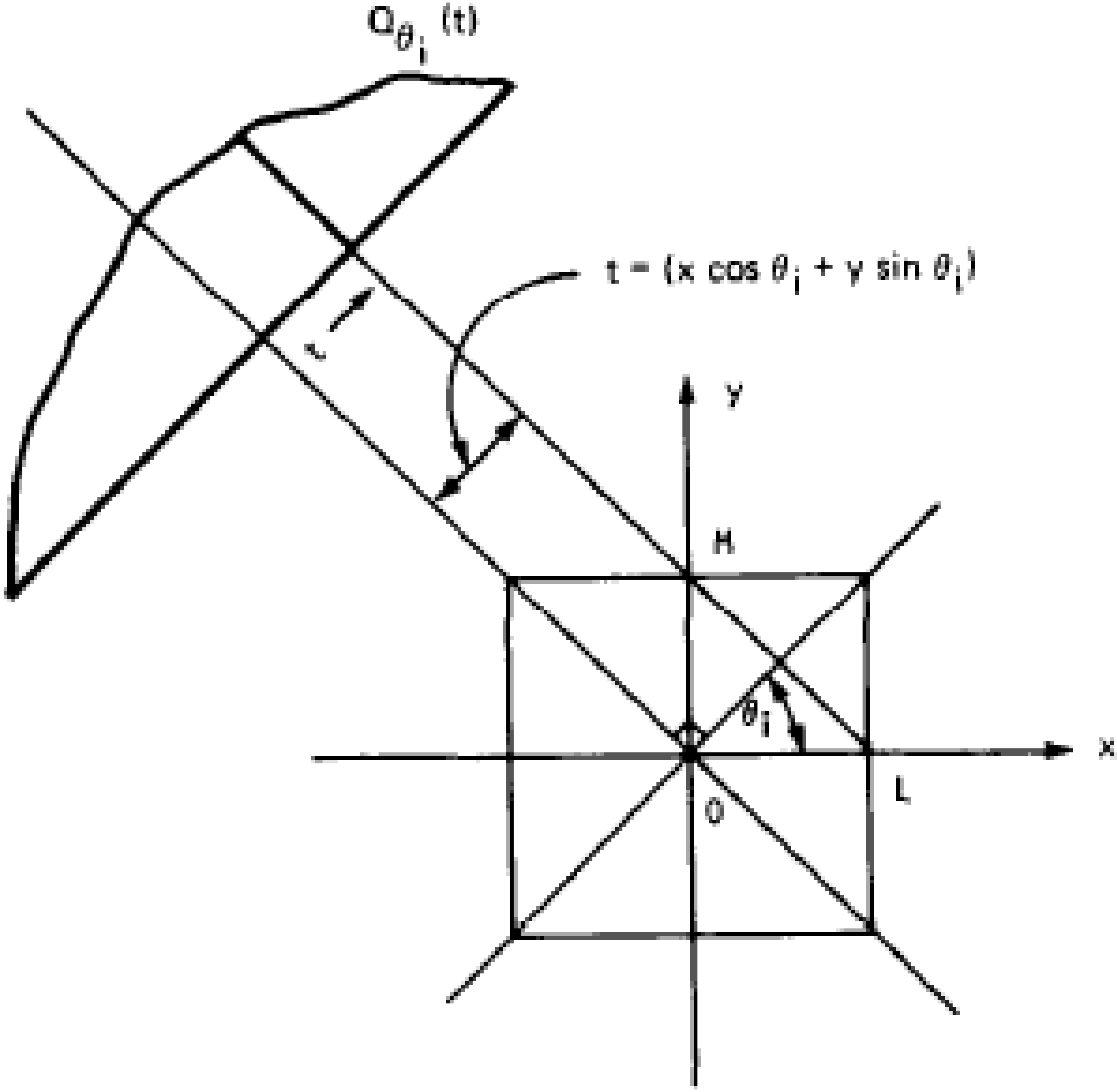
y
x
A
B
(x_1, y_1)
θ
α



projection
t
y
x
θ
object
Fourier transform
v
u
θ
B
B
space domain
frequency domain

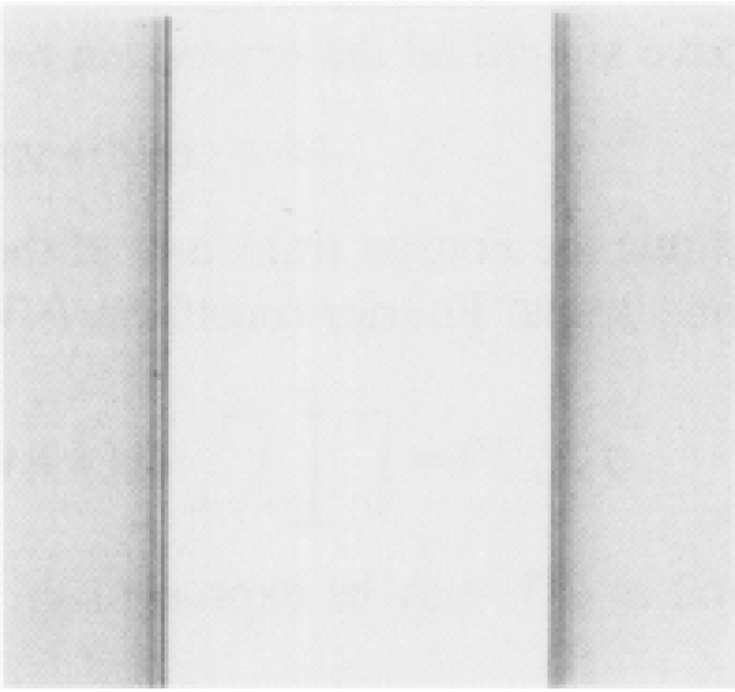


frequency domain

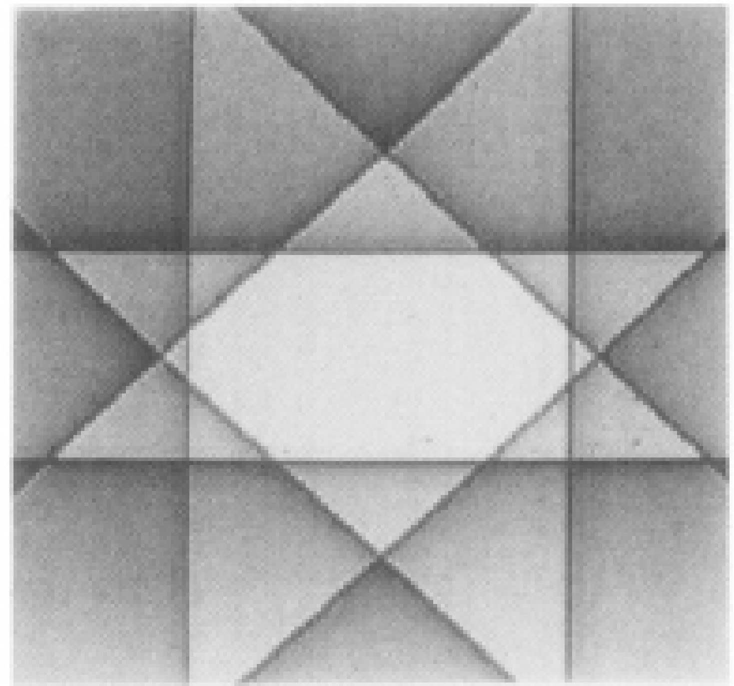


$Q_{\theta_i}(t)$
$t = (x \cos\theta_i + y \sin\theta_i)$
t
y
x
H
L
0
θ_i

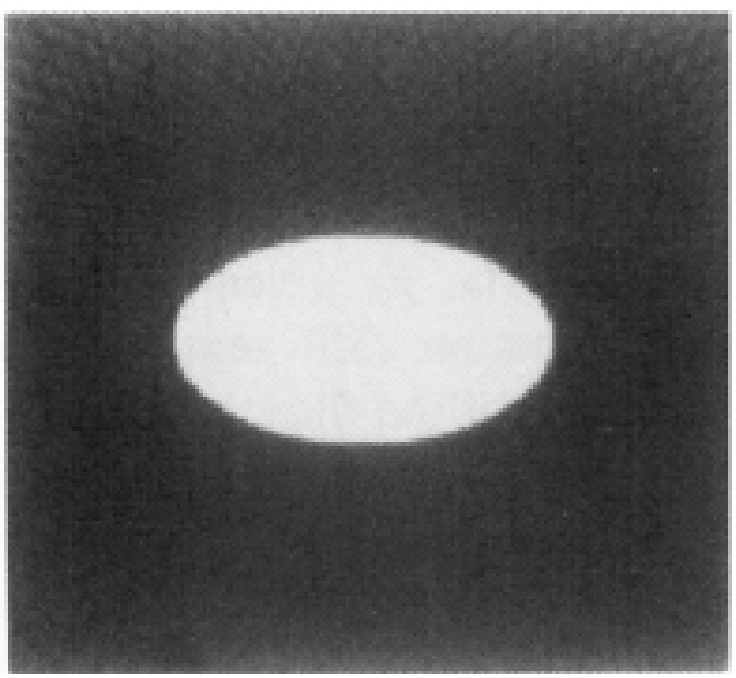
(a) 1 projection



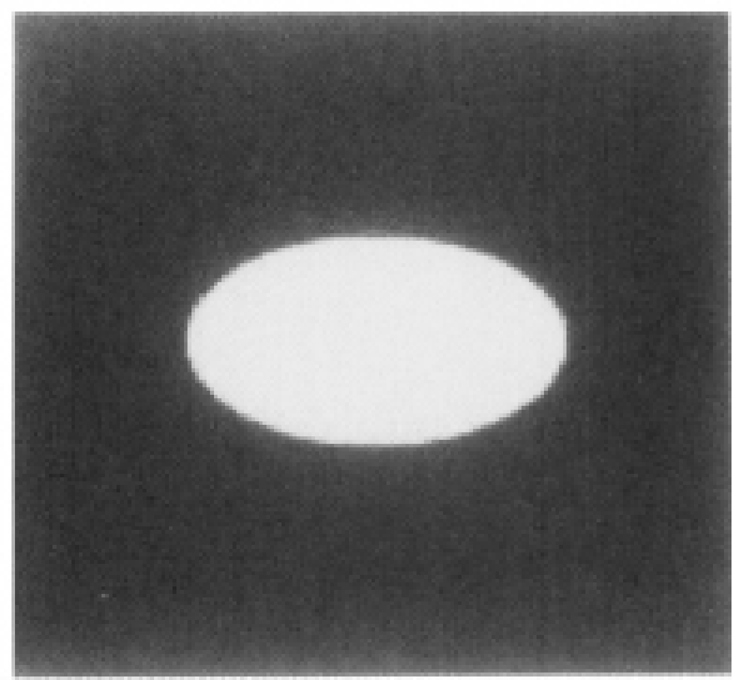
(b) 4 projections

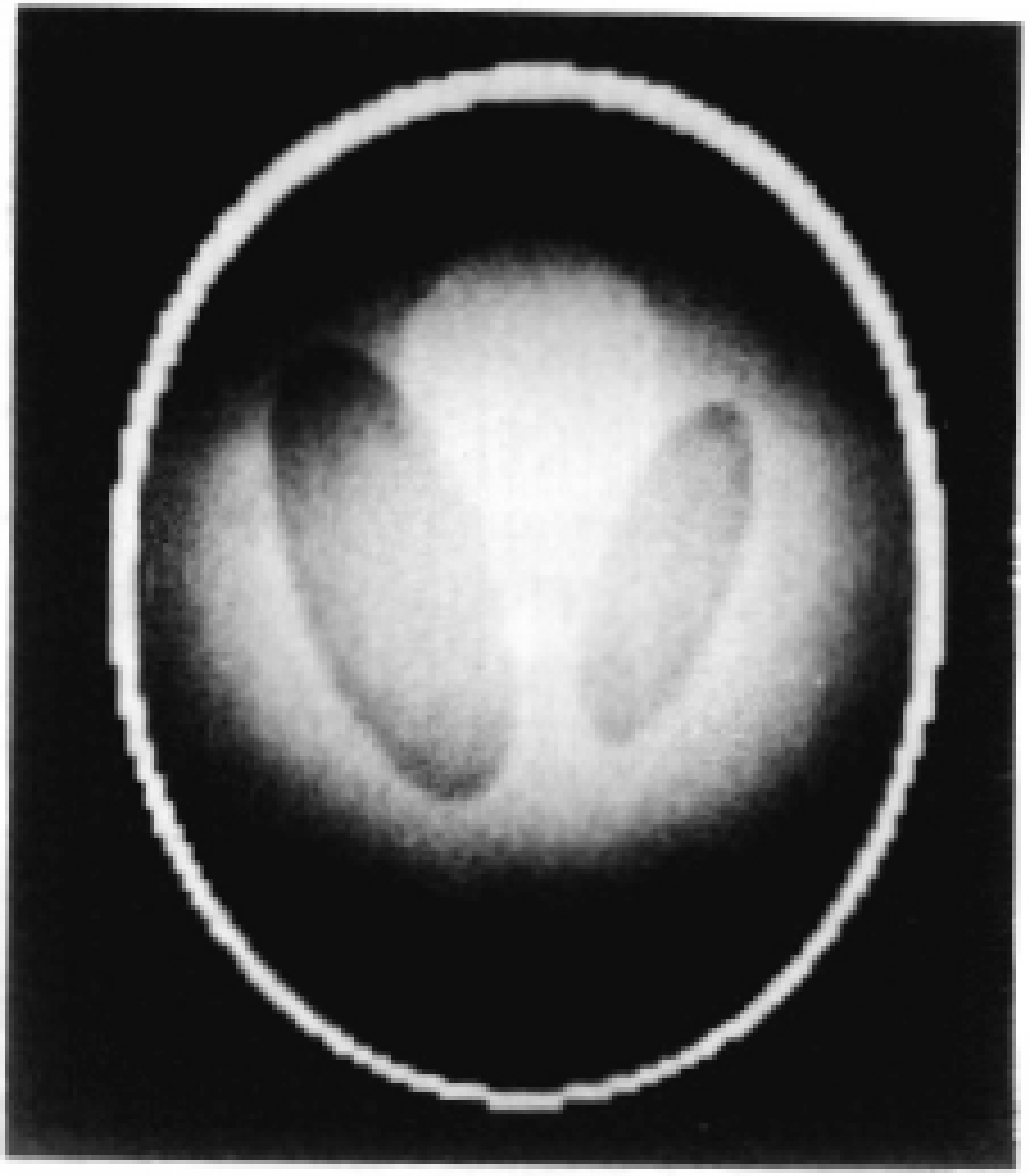


(c) 64 projections



(d) 512 projections





(a)