Material for Chapter 6:

Basic Principles of Tomography
Figure 1: Radon Transform: Attenuation

Parallel scanning geometry

Figure 2: Dual motion scanner. **Top:** One source, one detector. **Bottom:** One source, several detectors. Source: Natterer & Wübbeling, 2001.
Parallel scanning geometry II

Figure 3: Fan beam scanning. **Left:** Rotating detector-source system. **Right:** Stationary detector ring, rotating source. Source: Natterer & Wübbeling, 2001.
Figure 4: Object, $f(x, y)$, and its projection, $P_\theta(t_1)$, shown for angle $\theta$. Source: Kak, 1979.
Parallel Projections

Figure 5: Parallel projections are taken by measuring a set of parallel rays for a number of different angles. Source: Rosenfeld & Kak, 1982.
Fan Beam Projections

Figure 6: A fan beam projection is collected if all the rays meet in one location. Source: Rosenfeld & Kak, 1982.
For tests: head phantom

Figure 7: **Left:** The Shepp and Logan head phantom. **Right:** The phantom is a superposition of 10 ellipses. Source: Rosenfeld & Kak, 1982.
Example: Projected Ellipses

Figure 8: Analytic expression is shown for the projection of an ellipse. For computer simulations a projection can be generated by simply summing the projection of each individual ellipse. Source: Rosenfeld & Kak, 1982.
Example: Projected Ellipses II

Figure 9: Analytical calculation for an ellipse with its center located at \((x_1, y_1)\) and its major axis rotated by \(\alpha\). Source: Rosenfeld & Kak, 1982.
Fourier Slice Theorem

Figure 10: The Fourier Slice Theorem relates the Fourier transform of a projection to the Fourier transform of the object along a radial line. Source: Pan & Kak, 1983.
Interpolation in frequency domain ???

**Figure 11:** Collecting projections of the object at a number of angles gives estimates of the Fourier transform of the object along radial lines. The dots represent the actual location of estimates of the object’s Fourier transform. Source: Pan & Kak, 1983.
Backprojection I

Figure 12: Reconstructions are often done using a procedure known as backprojection. Here a filtered projection is smeared back over the reconstruction plane along lines of constant $t$. The filtered projection at a point $t$ makes the same contribution to all pixels along the line LM in the $x - y$ plane. Source: Rosenfeld & Kak, 1982.
Figure 13: The result of backprojecting the projection of an ellipse. (a) shows the result of backprojecting for a single angle, (b) shows the effect of backprojecting over 4 angles, (c) shows 64 angles, and (d) shows 512 angles.
Example: Reconstruction

Figure 14: Reconstruction of the head phantom. The dark regions at the top and at the bottom are visible artefacts. This $128 \times 128$ reconstruction was made from 110 projections with 127 rays in each direction. Source: Rosenfeld & Kak, 1982
Literature


\[ \vec{n} = (\cos \alpha, \sin \alpha) \]

\[ f(x, y) \]
\[ P_B(t) = \frac{2\rho AB}{a^2(\theta)} \sqrt{a^2(\theta) - t^2} \]

\[ a^2(\theta) = A^2 \cos^2 \theta + B^2 \sin^2 \theta \]

\[ f(x,y) = \rho \quad \text{inside ellipse} \]
\[ = 0 \quad \text{outside} \]
frequency domain
\[ t = (x \cos \theta_i + y \sin \theta_i) \]