Differential Geometric Aspects in Image Processing

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Problem C7.1

Let f and u be defined on a compact surface S without boundary.

i) Show that

$$\int_S f\Delta_S u dS = \int_S g(\nabla_S f, \nabla_S u)dS$$

Hint: You may assume that there exists a family of smooth nonnegative functions $\{\eta_i : S \to \mathbb{R}, 0 \leq i \leq M\}$ and an atlas $\{\phi_i : D_i \to S, 0 \leq i \leq M\}$ which satisfy: each $\eta_i \circ \phi_i$ has compact support of D_i and for any $x \in S$ there exists j s.t. $\eta_j(x) > 0$.

ii) Use the previous result to show that linear diffusion preserves the average grey value over the surface.

Problem C7.2

Let $\alpha_i : S \to \mathbb{R}$, $\alpha_i(x_1, x_2, x_3) = x_1$, for $1 \leq i \leq 3$. Show that

$$\sum_{i=1}^{3} ||\nabla_S \alpha_i||^2 = 2$$