# Differential Geometric Aspects in Image Processing

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## Problem C5.1

Let $\sigma : U \setminus l \to V \setminus L$ be a system of geodesic polar coordinates $(\rho, \theta)$ with $p \in U \subset T_pS$. Show that

i) $E = 1$

ii) $\Gamma_{11}^2 = 0$

iii) $\Gamma_{11}^1 = E_\rho = 0$ and $F_\rho = 0$

iv) $F = 0$

## Problem C5.2

Let

$$\alpha_t = N \times \overrightarrow{t^\alpha}, \quad \alpha(\cdot, 0) = \alpha_0(\cdot)$$

be the equal distance contour propagation around some $p \in S$, with $\alpha_0$ a geodesic circle. Moreover, let $u$ be s.t. $\{(x, y) : u(x, y) = t\}$ corresponds to the $2D$ projection of $\alpha(\cdot, t)$.

i) Show that $\Pi(\overrightarrow{t^\alpha}) = c(-u_y, u_x)$ for some $c \in \mathbb{R}$, where $\Pi(x, y, z) = (x, y)$.

ii) Show that $\overrightarrow{t^\alpha} = \frac{(-u_y, u_x, qu_x - pu_y)}{\sqrt{u_x^2 + u_y^2 + (qu_x - pu_y)^2}}$, with $(p, q) = (\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y})$