

Differential Geometric Aspects in Image Processing

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Problem C3.0

Describe a geometric construction of the level sets corresponding to the positive values of the distance function for a convex closed curve.

Problem C3.1

Show that a path connected regular oriented surface has umbilic points everywhere, then it is either contained in a plane or a sphere.

Problem C3.2

Let $f : \Omega \rightarrow \mathbb{R}$, $\Omega \subset \mathbb{R}^2$ be a greyscale image and consider the parametrised surface $\sigma : \Omega \rightarrow \mathbb{R}^3$

$$\sigma(u, v) = \left(\frac{f(u, v)}{\sqrt{2}} - \frac{u}{\sqrt{2}}, v, \frac{u}{\sqrt{2}} + \frac{f(u, v)}{\sqrt{2}} \right)$$

- i) Check that $\sigma(U)$ is a regular surface
- ii) Compute a Gauss map
- iii) Express the first and second fundamental form of σ in terms of derivatives of f .
- iv) Show that the Gaussian curvature $\mathbf{K}$ and the mean curvature $\mathbf{H}$ are given by

$$\mathbf{K} = \frac{f_{uu}f_{vv} - f_{vu}^2}{(1 + f_v^2 + f_u^2)^2}$$

$$\mathbf{H} = \frac{1}{2}(1 + f_v^2 + f_u^2)^{-\frac{3}{2}}((1 + f_v^2)f_{uu} - 2f_u f_v f_{uv} + (1 + f_u^2)f_{vv})$$