

Differential Geometric Aspects in Image Processing

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Problem C1.1

- i) Define an atlas for the ellipse $E := \left\{ (x, y)^\top \in \mathbb{R}^2 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \right\}$ and write down the change of coordinates (change of charts) for all intersecting pairs of charts.
- ii) Define an atlas of two charts for the circle, $a = b = 1$.

Problem C1.2

Compute the center of the osculating circle for:

- the graph of a function $f : [a, b] \rightarrow \mathbb{R}$ at each point $x \in]a, b[$, in terms of the first and second derivatives of f (you may assume that $|f''| > 0$).
- the ellipse of Problem C1.1 at the points where it intersects the x and y axis.

Problem C1.3

Let a circle c_0 with radius $r_0 > 0$ be given:

$$c_0 = (r_0 \cos p, r_0 \sin p)^\top, \quad p \in [-\pi, \pi]$$

and consider the evolution $c(p, t)$ of this initial curve under curvature motion. Determine the radius r of c as a function of t (you may assume because of symmetry that c is given by concentric circles).