Differential Geometric Aspects in Image Processing

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Problem H4.1 (6 Points)

1. Let L_i be the lines corresponding to the edges between z_k and z_l, with i, k, l all different indices and for $0 \leq i, k, l \leq 3$. The restriction of P to L_1 is a cubic polynomial of one variable with double roots at z_2 and z_3. Hence P is 0 along L_1. Similarly, P is 0 along the edges L_2 and L_3. By the lemma of Lecture 19 (slide 9), we can write $P = cL_1L_2L_3$, where c is a constant. However, $P(z_4) = 0$, which implies that $c = 0$, since all L_i are different from zero at the barycenter.

2.1 The dimension is given by $(k + 1)^2$

2.2 Analogous to point 1.

Problem H4.2 (6 Points)

Given a function $f \in C^2$, we may approximate it near $\mathbf{x_0} = (x_0, y_0)$ by its Taylor expansion of second order:

$$f(\mathbf{x}) = f(\mathbf{x}_0) + \nabla f(\mathbf{x}_0)(\mathbf{x} - \mathbf{x}_0)^\top + (\mathbf{x} - \mathbf{x}_0)H_f(\mathbf{x}_0)(\mathbf{x} - \mathbf{x}_0)^\top,$$

where H is the hessian of f. An approximation of a patch around $\mathbf{x}$ may thus be described by the values $\{f(\mathbf{x}_0), \nabla f(\mathbf{x}_0), H_f(\mathbf{x}_0)\}$. Therefore we may consider the family of functions $\{a + (b, c)(\mathbf{x} - \mathbf{x}_0)^\top + (\mathbf{x} - \mathbf{x}_0) \begin{pmatrix} f & g \\ g & h \end{pmatrix} (\mathbf{x} - \mathbf{x}_0)^\top \ : (a, b, c, f, g, h) \in \mathbb{R}^6\}$ for describing the patches, a manifold of dimension 6.