Differential Geometric Aspects in Image Processing

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Homework Assignment: January 31, 2020

Please submit your solutions on **February 11, 2020**

Remark: Always justify your answer! All steps of the solutions must be complete and consistent. Please do not submit electronically. Only handwritten solutions will be graded.

Problem H4.1 (6 Points)

1. Let $\mathcal{K}$ be a triangle with vertices z_1, z_2, z_3 and centroid z_4. Furthermore, let $\mathcal{P}$ be $\mathcal{P}_3$, the set of polynomials of degree less or equal than three.
Let $v \in \mathcal{P}_3$. Show that the nodal set $\mathcal{N} = \{\mathcal{N}_1, ..., \mathcal{N}_{10}\}$ given by the evaluation of v at points z_1, z_2, z_3, z_4 and of its partial derivatives at z_1, z_2, z_3, determines $\mathcal{P}_3$ (i.e. if $v \in \mathcal{P}_3$ and $\mathcal{N}_i(v) = 0$ for $i = 1, ..., 10$ then $v = 0$).

2. Let

$$Q_k := \{\sum_j c_j p_j(x) q_j(y) \ : \ p, q \text{ polynomials of deg} \leq k\}.$$

2.1 What is the dimension of the linear space Q_k?

2.2 Suppose that K is a rectangle, and let $\mathcal{P}$ be given by Q_2. Furthermore, let $\mathcal{N}$ be given by evaluation at the vertices, the centroid and the midpoints of the edges of the rectangle $\mathcal{K}$. Show that $\mathcal{N}$ determines $\mathcal{P}$.

Problem H4.2 (6 Points)

Let $\mathcal{F}$ be a family of $n \times m$ images obtained by samplings a subset of $C^2([0,1]^2)$. Consider the set of square patches $\{P_x \ : \ x \in f, f \in \mathcal{F}\}$ of size $p \times p$ centered at each point of the elements of $\mathcal{F}$ (eventually reflecting images for points near the boundaries).
1.Define a low dimensional manifold which approximately describes the set of patches considered as a set of points in $\mathbb{R}^{p \times p}$.