Differential Geometric Aspects in Image Processing

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Homework Assignment: December 5, 2019

Please submit your solutions on **December 12, 2019**

Remark: Always justify your answer! All steps of the solutions must be complete and consistent. Please do not submit electronically. Only handwritten solutions will be graded.

Problem H3.1 (6 Points)

i) Let v and w be vector fields along a curve $\alpha : I \to S$. Prove the following identity for the covariant derivatives

$$\frac{d}{dt}\langle v(t), w(t)\rangle = \left\langle \frac{Dv}{dt}, w(t)\right\rangle + \left\langle v(t), \frac{Dw}{dt}\right\rangle$$

ii) Let $a > r > 0$ and consider the torus obtained by rotating the circle $\{(x, y, z) \subset \mathbb{R}^3 : (x - a)^2 + z^2 = r^2, y = 0\}$ about the z axis. Which of the three curves obtained by rotating the points $(a + r, 0, 0), (a - r, 0, 0), (a, 0, r)$ about the z axis are geodesics.

Problem H3.2 (6 Points)

Let $\gamma : I \to \mathbb{R}^3$ be a unit speed curve with curvature $\kappa(t) \neq 0, \forall t \in I$. Let $\overrightarrow{b}(t)$ be the binormal of the curve at t. Put $\sigma(u, v) = \gamma(v) + u\overrightarrow{b}(v)$ for $(u, v) \in \mathbb{R} \times I$.

i) Show that σ is a regular parametrised surface ("the binormal surface of the curve")

ii) Show that γ is a geodesic