Differential Geometric Aspects in Image Processing

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Homework Assignment (Solutions): October 24, 2019

Problem H1.1 (4 Points)

i) Only c_3 is not regular and only c_4 is closed,

ii) c_1 corresponds to a portion of a circle with radius 1. For the rest we apply the general formula of the curvature:

$$\begin{aligned}
\kappa_1(p) &= 1 \\
\kappa_2(p) &= \frac{1}{(1+p^2)^{3/2}} \\
\kappa_3(p) &= \frac{2\pi\cos p\pi + (2p-1)\pi^2 \sin p\pi}{\left((\pi^2\cos^2 p\pi + (2p-1)^2\right)^{3/2}} \\
\kappa_4(p) &= \frac{120}{\left(36\sin^2 2p + 100\cos^2 2p\right)^{3/2}}
\end{aligned}$$

Problem H1.2 (4 Points)

i) Corresponds to erosion since the curve flow is given by

$$c_t(p,t) = (-\cos p, -\sin p) = \overrightarrow{n}(p,t)$$

ii) Integrating w.r.t. t we obtain that

$$c(p,T) = c(p,0) + \int_0^T c_t(p,t)\,dt =$$

$$\left((r_0+T)\cos p - \frac{T^2}{2}\sin p, (r_0+T)\sin p + \frac{T^2}{2}\cos p\right).$$

Therefore

$$||c(p,T)|| = \sqrt{(r_0+T)^2 + \frac{T^4}{4}} \tag{1}$$

and its graph $\{c(p,T) : p \in [0,2\pi]\}$ is the circle centered at the origin with radius given by the r.h.s. of (1).

Problem H1.3 (4 Points)

By symmetry the total length is $L(c) = 2L(c^+)$, where c^+ is the the part of the curve c with graph in the half space $\{z > 0\}$. Using the parametrisation $c^+(p) = (p, p, \sqrt{f(p)})$, for $-1 \le p \le 1$, we obtain

$$L(c) = 2\int_{-1}^{1} \sqrt{2 + (f'(p))^2}\, dp.$$