Differential Geometric Aspects in Image Processing

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Homework Assignment: October 24, 2019

Please submit your solutions on **October 31, 2019, in the lecture break.**

Remark: Always justify your answer! All steps of the solutions must be complete and consistent. Please do not submit electronically. Only handwritten solutions will be graded.

Problem H1.1 (4 Points)

Consider the curves:

$$\begin{aligned} c_1(p) &= (\cos p/2, \sin p/2) && p \in [0, \pi] \\ c_2(p) &= (p, p^2/2) && p \in [-1, 1] \\ c_3(p) &= (\sin p\pi, (p - 1/2)^2) && p \in [0, \frac{2}{3}] \\ c_4(p) &= (3\cos 2p, 5\sin 2p) && p \in [0, \pi] \end{aligned}$$

i) Which of the curves are regular? Which are closed?
ii) Compute the curvatures as a function of p.
iii) Find the reparametrisations $s = \psi(p)$ that make $c_1(s)$ and $c_2(s)$ into arc-length parametrised curves.

Problem H1.2 (4 Points)

i) Consider the curve evolution

$$c(p, t) = ((1 - t)\cos p, (1 - t)\sin p) \quad (p, t)^\top \in [0, 2\pi] \times [0, 1].$$

What is the curve flow of c? Does it correspond to erosion, dilation or curvature flow?

ii) Consider the curve evolution given by

$$\begin{aligned} c_t &= (\cos p - t \sin p, \sin p + t \cos p) && p \in [0, 2\pi], \\ c(p, 0) &= (r_0 \cos p, r_0 \sin p) && p \in [0, 2\pi]. \end{aligned}$$

What is the graph of the curve $c(p, T)$ if T is any positive value?

Problem H1.3 (4 Points)

i) Let $f : [-1, 1] \to \mathbb{R}$ be a smooth nonnegative function and let

$$\{(x, y, z)^\top \in \mathbb{R}^3 \,:\, x = y,\, -1 \leq x \leq 1\} \cap \{(x, y, z)^\top \in \mathbb{R}^3 \,:\, z^2 = f(x)\}$$

be the graph of a simple closed curve. Write the length of this curve as a function of f'.

Hint: Use the symmetry w.r.t. the xy plane and write the curve as a function of x.