Variational Deblurring of Images with Uncertain and Spatially Variant Blurs

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Abstract. We consider the problem of deblurring images which have been blurred by different reasons during image acquisition. We propose a variational approach admitting spatially variant and irregularly shaped point-spread functions. By involving robust data terms, it achieves a high robustness particularly with respect to imprecisions in the estimation of the point-spread function. A good restoration of image features is ensured by using non-convex regularisers and a strategy of reducing the regularisation weight. Experiments with irregular spatially invariant as well as with spatially variant point-spread functions demonstrate the good quality of the method as well as its stability under noise.
Keywords: deblurring, variational method, robust data terms

1 Introduction

Blurring of images in the process of acquisition appears in many application contexts. Reasons include defocussing as well as camera and object motion during exposition. Besides that, optical imperfection of the lens system also causes blurring. Ubiquity of these image degradation makes deblurring a problem of outstanding importance in image processing.

Often the blurring can be captured by convolution of a sharp image with some kernel, or point-spread function (PSF), which does not depend on the location within the image. Typical cases where such *spatially invariant* blur occurs are translatory motion of the camera, and defocussing if all objects are at equal depth. However, in many important cases the PSF changes between different locations in the image, i.e. one has a *spatially variant* blur. Even rotations of the camera for static scenes lead to spatially variant blur.

Assume first we have spatially invariant blur with known PSF. One relatively simple approach to deblurring is then to multiply the Fourier transform of the blurred image by the reciprocal $\hat{h}^{-1}$ of the Fourier transform of the kernel h. Refinement of this idea by suitable treatment of such frequencies for which $\hat{h}$ is close to zero leads to classical linear deblurring filters like pseudoinverse filtering and Wiener filtering [21, 10]. Limitations of this approach are its restriction to spatially invariant blur as well as characteristic oscillatory artifacts which cannot be avoided by linear methods [3].

The variational deblurring approach is able to reduce these artifacts. A rich literature exists on variational deblurring in the case of spatially invariant PSF. The choice of suitable regularisation terms ensures that important structures in the image like edges are restored in good quality. In this paper, we will extend the variational approach to a slightly more general form which also covers spatially variant blurs.

It is worth noting that many papers on deblurring show only synthetic tests where the degraded image is obtained by blurring a sharp image with a known PSF. In application contexts, however, the PSF will mostly not be known exactly. In some cases, such as optical errors, it can be measured fairly precise in calibration procedures. In other cases a-priori knowledge restricts the variety of possible kernels, e.g. in defocussing where typically cylindrical kernels occur. Sometimes no restricting assumptions of this kind hold.

As a remedy for this deficient knowledge on the PSF, one can consider blind deconvolution which estimates a convolution kernel and sharp image at the same time. Variational blind deconvolution methods can be found e.g. in [22, 9]. For the ill-posedness of the deblurring problem, however, it is desirable to introduce as much a-priori information as one can have into the deblurring process. The approach we present here is a non-blind deconvolution method which is tolerant to imprecision in the kernel to a higher degree than previous ones. In fact, it allows a reasonable deblurring of images which are degraded by natural sources of blur and for which we do principally not know a precise PSF or a sharp ground-truth image. The PSFs in our experiments are just approximated to a modest precision from manually selected image features.

The key to this enhanced robustness is to employ in the variational ansatz *robust data terms* which made their first appearance in a deblurring context in the paper by Bar et al. [2]. Robust data terms are motivated by robust statistics [13, 11] and have been introduced in computer vision particularly in motion detection [5, 6, 12, 16, 8]. Important theoretical results were contributed by Nikolova [19].

Our paper is organised as follows. In Sec. 2 we derive the gradient descent PDE with robust data terms and spatially invariant PSF. Subsection 2.2 is devoted to specific problems of the selection of the regularisation weight and treatment of boundaries. Experiments are presented in Sec. 3. We end with conclusions in Sec. 4.

Related work. The deblurring problem has played a role in computer vision research over a considerable time. A great variety of approaches have been undertaken. Blind and non-blind variational deconvolution with total variation regularisation has been considered in [15] as well as in [9] and [1], the latter in combination with a segmentation approach. A blind deconvolution approach with more general regulariser is found in [22]. Bar et al. [2] introduced robust data terms into this field. Bertero et al. [4] contributed results on existence, uniqueness and stability of solutions for variational problems of this type.

Deconvolution with spatially variant PSF has been studied mainly within discrete frameworks such as [17, 18, 14].

2 Variational Deblurring with Spatially Variant PSF

2.1 Basic Model with Robust Data Terms

A general PSF has the form $H(y,x)$ where y, x denote locations in the sharp and degraded images, respectively. The sharp image g and degraded image f are then connected via

$$f(x) = \int_{\Omega} g(y)\, H(y,x)\, \mathrm{d}y + n(x)$$

where $\Omega \subset \mathbb{R}^2$ is the image domain, and n denotes noise. Given some approximation u for g, we have the *residual error*

$$R_{f,H}[u](x) := f(x) - \int_{\Omega} u(y)\, H(y,x)\, \mathrm{d}x\ .$$

Variational deblurring of the image f is achieved by minimising the functional

$$E(u) = \int_{\Omega} \Big(\varPhi((R_{f,H}[u])^2) + \alpha \varPsi(|\nabla u|^2) \Big)\ \mathrm{d}x \tag{1}$$

where $\varPhi$ and $\varPsi$ are monotonically increasing functions from $\mathbb{R}_0^+$ to $\mathbb{R}$. The first summand in the integrand is the *data term* which favours images u with small residual error. The second summand, called *regulariser*, enforces smoothness of the image. The strength of its influence is determined by the *regularisation weight* $\alpha > 0$. If the function $\varPhi$ increases for $s \to \infty$ slower than $\varPhi(s^2) = s^2$, one speaks of a *robust data term* since in this case large residual errors are given less influence on the value of $E(u)$ than with the quadratic error term. One typical choice is the regularised L^1-norm $\varPhi(s^2) = \sqrt{s^2 + \beta^2}$ with $\beta > 0$.

Robust data terms have been introduced in a deblurring context only recently by Bar et al. [2]. In the regularisation term, (regularised) total variation $\varPsi(s^2) = \sqrt{s^2 + \varepsilon^2}$ (with small positive ε) has already been preferred over the quadratic Tikhonov regulariser $\varPsi(s^2) = s^2$ for a long time because of their better ability to preserve sharp edges in the image. The non-convex Perona–Malik regulariser $\varPsi(s^2) = (1 + s^2/\lambda^2)^{-1}$ has been considered in [20]. Here, $\lambda > 0$ is a contrast parameter which determines above which steepness edges are enhanced in the gradient descent process.

A gradient descent equation for (1) reads

$$\partial_t u = \alpha \operatorname{div}(\varPsi'(|\nabla u|^2)\, \nabla u) - \int_{\Omega} \varPhi'((R_{f,H}[u])^2)(y) \cdot R_{f,H}[u](y) H(y,x)\, \mathrm{d}y \tag{2}$$

which can be evaluated numerically in the simplest case by an explicit scheme. A speed-up could be achieved by using more efficient semi-implicit schemes.

2.2 Boundary Treatment and Parameter Choice

Notice that when writing spatially invariant blur as convolution, problems occur at the boundaries where the blur transfers information from inside the image to outside and vice versa. Our formalism with a general PSF $H(y, x)$ with two location arguments removes these difficulties since only locations within the image occur. Transfer of image information from inside to outside is neglected while the reverse transfer can be considered as part of the noise.

To be able to choose appropriately the parameter α, we must be aware of the double function of the regularisation in deblurring. On one side, deblurring is a highly ill-posed problem such that regularisation is needed to suppress noise. This entails that in the presence of strong noise α should be chosen larger. On the other side, even in a noise-free setting the regularisation term has the function to suppress certain oscillatory perturbations in the filtered image which are not detected by the data term.

The latter aspect generally leads to a much larger α than the pure noise suppression. However, such strong regularisation introduces an unwanted smearing of structures into the filtered image. In [20] a continuation strategy has been proposed which consists in starting with large α and repeating the gradient descent several times with successively decreasing α. In each decrement step, the previously reached steady state serves as initialisation. This method combines a good suppression of oscillation with sharp restoration of image structures. A similar continuation strategy had been proposed in a different context in [7]. We will use this continuation strategy also here. Here, the initial level of α is primarily adapted to the suppression of oscillations while the final level of α depends on the noise intensity.

3 Experiments

In our tests, we used the gradient descent PDE (2) with the robust data term $\Phi(s^2) = \sqrt{s^2 + \beta^2}$ (except for the non-robust case in Fig. 1 (c)). Tieing up with [20] we chose a Perona–Malik regulariser in all cases. With our present implementation based on an explicit scheme, computation times range from 15 to 90 minutes on standard PCs for the images shown in this paper.

Our first experiment, Fig. 1, shows a photograph of Paris at dusk blurred by camera movement during the exposition, with three enlarged detail views. The resulting PSF is fairly irregular. For the deblurring process, it was assumed to be spatially invariant. The PSF then was approximated by clipping the image of an isolated light source from the lower part of the river region. Clearly, the estimation of the convolution kernel from such an approximate impulse response induces an imprecision. Further, a closer look at the blurred image reveals that the assumption of spatially invariant blur does not perfectly capture the situation since impulse responses in opposite corners of the image are of slightly different shape. In the case of non-robust variational deblurring it was therefore necessary to choose a large diffusion weight which caused most finer structures in the image to be smoothed away. Nevertheless, street lights are still restored inaccurately, as they are accompanied by shadows and echo images. Street lights far from the river region are badly restored because of the PSF inaccuracy.

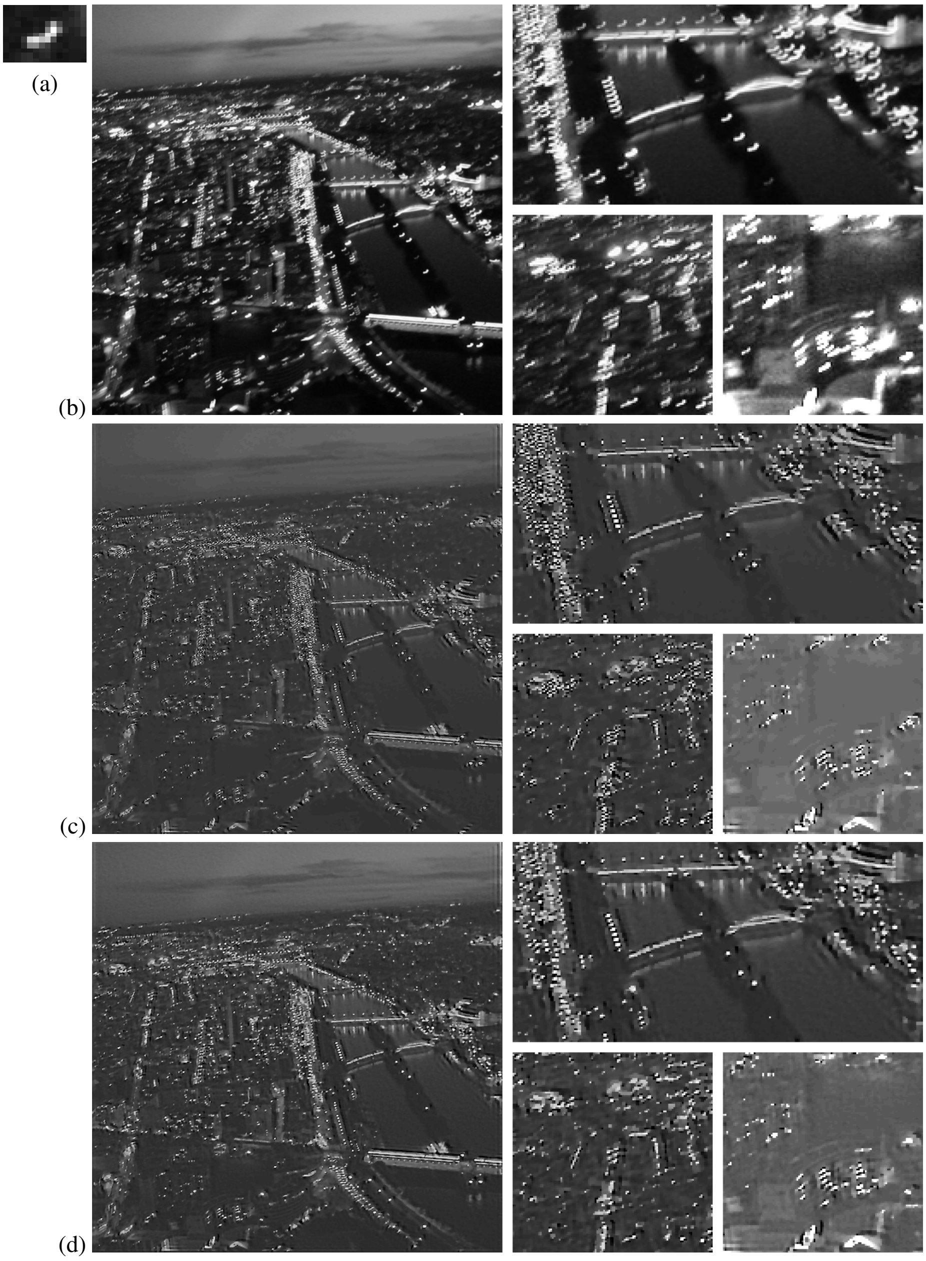
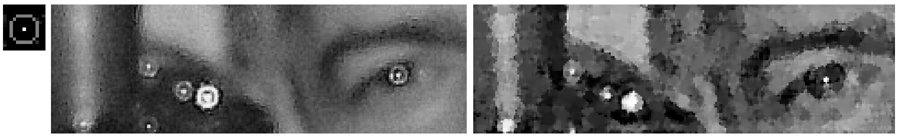


Fig. 1. (a) Spatially invariant, irregularly shaped approximate PSF for deconvolution (enlarged). – **(b)** Paris at dusk from Eiffel tower, blurred by camera movement during exposition. Complete image (480×480) and three detail views. Grey-values in lower right detail view have been linearly rescaled. – **(c)** Restoration by variational deblurring with non-robust data term and Perona–Malik regulariser, $\lambda = 15$, $\alpha = 0.1$. – **(d)** Same with robust data term, $\lambda = 15$, $\alpha = 0.02$.



(a) (b) (c)

Fig. 2. (a) Spatially invariant approximate PSF (enlarged). – **(b)** Blurred photograph (clipping). – **(c)** Variational deblurring with robust data terms, Perona–Malik regularisation ($\lambda = 40$) and continuation strategy (4 levels with $\alpha = 0.06, 0.03, 0.015, 0.0075$).

Robust deblurring, in contrast, can cope with the PSF imprecision much better. A smaller diffusion weight has been chosen such that more structures like streets and buildings are recovered. Shadows and double images near the street lights do hardly occur. Even in the lower left part of the image where the PSF's shape deviates much from that in the river region favourable sharpness is achieved.

Secondly, we show a detail from a photograph where defocussing together with possible optical errors have introduced a ring-like blur, Fig. 2. The PSF has again been distilled from an approximate impulse response in the image. Obviously it is only a rough estimate. Moreover the image contains many half-tones. The filtered image displays a trend towards steepened contrasts but besides that reveals a sharpening of small structures and highlights.

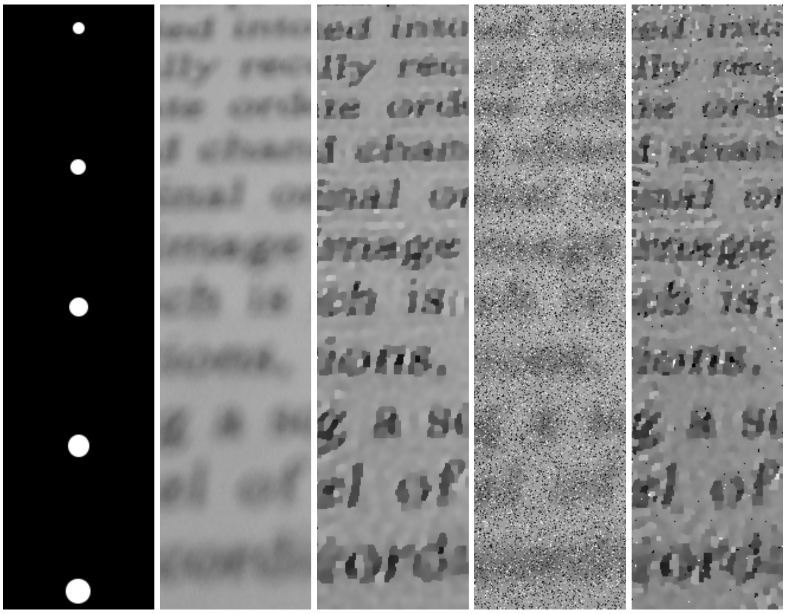
In our last experiment, Fig. 3, a piece of printed text was photographed from small distance without appropriate focussing. The distance between lens and object varied widely, leading to a stronger defocussing in the lower than in the upper part of the image. Theoretical considerations show that defocussing PSFs are well approximated by cylindrical functions. For defocussing we used therefore a cylindrical PSF whose diameter varies linearly from 5 at the top edge to 10.5 at the bottom edge.

To demonstrate also the robustness of the proposed deblurring method with respect to noise, we replaced $30\,\%$ of all pixels by uniform noise. Noise of such intensity is not typically encountered in application data that the method is intended for, so we had to resort to artificial image degradation at this single point. Restoration using our variational method still works well. One difference is that the continuation strategy must stop reducing the diffusion weight α at a larger value now, in order to remove the oversmoothing at the initial large α while preserving the noise suppression by regularisation.

We stress that although we used in Fig. 3 a special PSF shape where effectively only one parameter – the diameter – controls the spatial variation, our deblurring PDE itself is not restricted to such a setting. Indeed it can cope with arbitrarily given $H(y, x)$.

4 Conclusions and Ongoing Work

We have proposed a general energy-minimising approach to image deblurring which allows to treat spatially variant blurs and includes the robust data terms which have first been used in this context by Bar et al. [2]. We used a non-convex regulariser of Perona–Malik type. By employing also the continuation strategy for the regularisation weight that was previously developed in [20], we have obtained a performant method.



(a) (b) (c) (d) (e)

Fig. 3. (a) Spatially variant defocussing PSF (correct size). – **(b)** Defocussed photograph of printed text. – **(c)** Restored by variational deblurring with robust data term and Perona–Malik regulariser ($\lambda = 5$), using continuation strategy (2 levels with $\alpha = 0.03$ and $\alpha = 0.003$). – **(d)** Defocussed photograph with $30\,\%$ uniform noise. – **(e)** Restored by variational deblurring with robust data term and Perona–Malik regulariser ($\lambda = 5$), using continuation strategy (2 levels with $\alpha = 0.03$ and $\alpha = 0.015$).

Experiments demonstrate that image blurs of fairly general shape and different types which emerged during image acquisition could be reverted by this method.

Ongoing work aims at the development of methods to automatise the detection of point-spread functions as well as improved representation of more general types of spatially variant PSFs.

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