Coherence-Enhancing Shock Filters

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Abstract. Shock filters are based in the idea to apply locally either a dilation or an erosion process, depending on whether the pixel belongs to the influence zone of a maximum or a minimum. They create a sharp shock between two influence zones and produce piecewise constant segmentations. In this paper we design specific shock filters for the enhancement of coherent flow-like structures. They are based on the idea to combine shock filtering with the robust orientation estimation by means of the structure tensor. Experiments with greyscale and colour images show that these novel filters may outperform previous shock filters as well as coherence-enhancing diffusion filters.

1 Introduction

Shock filters belong to the class of morphological image enhancement methods. Most of the current shock filters are based on modifications of Osher and Rudin's formulation in terms of partial differential equations (PDEs) [12]. Shock filters offer a number of advantages: They create strong discontinuities at image edges, and within a region the filtered signal becomes flat. Thus, shock filters create segmentations. Since they do not increase the total variation of a signal, they also possess inherent stability properties. Moreover, they satisfy a maximum–minimum principle stating that the range of the filtered image remains within the range of the original image. Thus, in contrast to many Fourier- or wavelet-based strategies or linear methods in the spatial domain [19], over- and undershoots such as Gibbs phenomena cannot appear. This makes shock filters attractive for a number of applications where edge sharpening and a piecewise constant segmentation is desired. Consequently, a number of interesting modifications of the original schemes has been proposed [1, 5, 9, 11, 17]. All these variants, however, still pursue the original intention of shock filtering, namely edge enhancement.

Diffusion filters constitute another successful class of PDE-based filters [14, 20]. Compared to shock filters, diffusion filters have stronger smoothing properties, which may be desirable in applications where noise is a problem. While many diffusion filters act edge-enhancing, there are also so-called coherence-enhancing diffusion filters [21, 22]. They are designed for the enhancement of oriented, flow-like structures, appearing e.g. in fingerprint images. The basic idea is to diffuse anisotropically along the flow field such that gaps can be closed. A number of variants exist that have been applied to crease enhancement [18], seismic imaging [7] or flow visualisation [15].

In some of these application areas, noise is not a severe problem. Then the smoothing properties of coherence-enhancing diffusion are less important, while it would be desirable to have stronger sharpening qualities. A first step in this direction was pursued by a filter by Kimmel et al. [8], where backward diffusion is used. Although the results look impressive, the authors mention instabilities caused by the backward diffusion process. Thus the filter could only be used for short times and favourable stability properties as in the case of shock filtering cannot be observed.

The goal of the present paper is to address this problem by proposing a novel class of shock filters, so-called *coherence-enhancing shock filters*. They combine the stability properties of shock filters with the possibility of enhancing flow-like structures. This is achieved by steering a shock filter with the orientation information that is provided by the so-called structure tensor [2, 4, 16]. As a result, our novel filter acts like a contrast-enhancing shock filter perpendicular to the flow direction, while it creates a constant signal along the flow direction by applying either a dilation or an erosion process.

Our paper is organised as follows. In Section 2 we review some important aspects of shock filtering, and Section 3 describes the structure tensor as a tool for reliable orientation estimation. Both ingredients are combined in Section 4, where we introduce coherence-enhancing shock filters. Numerical aspects are briefly sketched in Section 5. In Section 6 we present a number of experiments in which the qualities of coherence-enhancing shock filtering are illustrated. Section 7 concludes the paper with a summary.

2 Shock Filters

Already in 1975, Kramer and Bruckner have proposed the first shock filter [10]. It is based on the idea to use a dilation process near a maximum and an erosion process around a minimum. The decision whether a pixel belongs to the influence zone of a maximum or a minimum is made on the basis of the Laplacian. If the Laplacian is negative, then the pixel is considered to be in the influence zone of a maximum, while it is regarded to belong to the influence zone of a minimum if the Laplacian is positive. Iterating this procedure produces a sharp discontinuity (shock) at the borderline between two influence zones. Within each zone, a constant segment is created. Iterated shock filtering can thus be regarded as a morphological segmentation method. The method of Kramer and Bruckner has been formulated in a fully discrete way.

The term *shock filtering* has been introduced by Osher and Rudin in 1990 [12]. They proposed a continuous class of filters based on PDEs. The relation of these methods to the discrete Kramer–Bruckner filter became evident several years later [6, 17]. To explain the idea behind shock filtering, let us consider a continuous image $f : \mathbb{R}^2 \to \mathbb{R}$. Then a class of filtered images $\{u(x, y, t) \,|\, t \geq 0\}$ of $f(x, y)$ may be created by evolving f under the process

$$u_t = -\text{sign}(\triangle u)\, |\nabla u|, \tag{1}$$

$$u(x, y, 0) = f(x, y), \tag{2}$$

where subscripts denote partial derivatives, and $\nabla u = (u_x, u_y)^\top$ is the (spatial) gradient of u. The initial condition (2) ensures that the process starts at time $t = 0$ with the original image $f(x, y)$. The image evolution proceeds in the following way: Assume that some pixel is in the influence zone of a maximum where its Laplacian $\triangle u := u_{xx} + u_{yy}$ is negative. Then (2) becomes

$$u_t = |\nabla u|. \tag{3}$$

Evolution under this PDE is known to produce at time t a dilation process with a disk-shaped structuring element of radius t; see e.g. [3]. At the influence zone of a minimum with $\triangle u < 0$, equation (2) can be reduced to an erosion equation with a disk-shaped structuring element:

$$u_t = -|\nabla u|. \tag{4}$$

These considerations show that for increasing time, (1) increases the radius of the structuring element until it reaches a zero-crossing of $\triangle u$, where the influence zones of a maximum and a minimum meet. Thus, the zero-crossings of the Laplacian serve as an edge detector where a shock is produced that separates adjacent segments. The dilation or erosion process ensures that within one segment, the image becomes piecewise constant.

A number of modifications have been proposed in order to improve the performance of shock filters. For instance, it has been mentioned in [12] that the second directional derivative $u_{\eta\eta}$ with $\eta \parallel \nabla u$ can be a better edge detector than $\triangle u$. In order to make the filters more robust against small scale details, Alvarez and Mazorra [1] replaced the edge detector $u_{\eta\eta}$ by $v_{\eta\eta}$ with $v := K_\sigma * u$. In this notation, K_σ is a Gaussian with standard deviation σ, and $*$ denotes convolution. Taking into account these modifications the shock filter becomes

$$u_t = -\text{sign}(v_{\eta\eta})\, |\nabla u|. \tag{5}$$

3 The Structure Tensor

It is not surprising that the performance of the shock filter (5) strongly depends on the direction η. Unfortunately, in the presence of flow-like structures (e.g. fingerprints) it is well known that the gradient of a Gaussian-smoothed image $K_\sigma * u$ does not give reliable information on the orientation, since parallel lines lead to patterns with opposite gradients [21]. Smoothing them over a window leads to cancellation effects, such that the resulting gradient direction shows very large fluctuations. To circumvent this cancellation problem, a more reliable descriptor of local structure is needed. To this end we replace ∇u by its tensor product

$$J_0(\nabla u) = \nabla u \, \nabla u^\top. \tag{6}$$

This matrix gives the same result for gradients with opposite sign, since $J_0(\nabla u) = J_0(-\nabla u)$. Now it is possible to average orientations by smoothing $J_0(\nabla u)$ componentwise with a Gaussian of standard deviation ρ:

$$J_\rho(\nabla u) = K_\rho * (\nabla u \, \nabla u^\top). \tag{7}$$

This 2×2 matrix is called *structure tensor (second-moment matrix, scatter matrix, Förstner interest operator)*; see e.g. [2, 4, 16]. It is positive semidefinite, and its orthonormal system of eigenvectors describes the directions where the local contrast is maximal resp. minimal. This contrast is measured by its eigenvalues.

Let w be the normalised eigenvector corresponding to the largest eigenvalue. In the following we shall call w the *dominant eigenvector* of J_ρ. In a flow-like pattern such as a fingerprint it describes the direction where the contrast change is maximal. This is orthogonal to the orientation of the fingerprint lines.

4 Coherence-Enhancing Shock Filtering

Now we are in the position to apply our knowledge about the structure tensor for designing novel shock filters. To this end, we replace the shock filter (5) by

$$u_t = -\operatorname{sign}(v_{ww}) \, |\nabla u| \tag{8}$$

where $v = K_\sigma * u$, and w is the normalised dominant eigenvector of the structure tensor $J_\rho(\nabla u)$. The direction w guarantees that this model creates shocks *orthogonal* to the flow direction of the pattern. In this *shock direction*, contrast differences are maximised. Along the perpendicular *flow direction*, either dilation or erosion takes place. Thus, after some time, structures become constant along the flow direction, and sharp shocks are formed orthogonal to it. Experimentally one observes that after a *finite* time t, the evolution reaches a piecewise constant segmentation where coherent, flow-like patterns are enhanced. Thus it is not required to specify a stopping time.

The *structure scale* σ determines the size of the resulting flow-like patterns. Increasing σ gives an increased distance between the resulting flow lines: Typically one obtains line thicknesses in the range of 2σ to 3σ. Often σ is chosen in the range between 0.5 and 2 pixel units. It is the main parameter of the method and has a strong impact on the result.

The *integration scale* ρ averages orientation information. Therefore, it helps to stabilise the directional behaviour of the filter. In particular, it is possible to close interrupted lines if ρ is equal or larger than the gap size. In order to enhance coherent structures, the integration scale should be larger than the structure scale. One may couple ρ to σ e.g. by setting $\rho := 3\sigma$. Since overestimations are uncritical, setting ρ to a fixed value such as $\rho := 5$ is also a reasonable choice.

The simplest way to perform coherence-enhancing shock filtering on a *multichannel image* $(f_1(x, y), ..., f_m(x, y))^\top$ consists of applying the process channelwise. Since this would create shocks at different locations for the different channels, some synchronisation is desirable. Therefore, we use the PDE system

$$u_{it} = -\text{sign}(v_{ww})\,|\nabla u_i| \qquad (i = 1, ..., m) \tag{9}$$

where $v_{ww} := \sum_{i=1}^{m} v_{iww}$, and w is the normalised dominant eigenvector of the joint structure tensor $J_\rho(\nabla u) := K_\rho * \sum_{i=1}^{m} \nabla u_i\, \nabla u_i^\top$. Similar strategies are used for coherence-enhancing diffusion of multichannel images [22]. Within finite time, a piecewise constant segmentation can be observed where the segmentation borders are identical for all channels.

5 Discretisation

For the algorithmic realisation of our shock filter, Gaussian convolution is approximated in the spatial domain by discretising the Gaussian, truncating it at tree times its standard deviation and renormalising it such that the area under the truncated Gaussian sums up to 1 again. Exploiting the separability and the symmetry of the Gaussian is used for speeding up the computations.

For the structure tensor, spatial derivatives have been approximated using Sobel masks. Since the structure tensor is a 2×2 matrix, one can easily compute its eigenvalues and eigenvectors in an analytical way.

If $w = (c, s)^\top$ denotes the normalised dominant eigenvector, then v_{ww} is computed from $c^2 v_{xx} + 2cs v_{xy} + s^2 v_{yy}$, where the second-order derivatives v_{xx}, v_{xy} and v_{yy} are approximated by standard finite difference masks.

For computing the dilations and erosions, an explicit Osher-Sethian upwind scheme is used [13]. This algorithm is stable and satisfies a discrete maximum–minimum principle if the time step size restriction $\tau \leq 0.5$ is obeyed. Thus, our shock filter cannot produce any over- and undershoots.

6 Experiments

We start our experimental section by comparing the difference between the conventional shock filter (5) and coherence-enhancing shock filtering. This is illustrated with the fingerprint image in Figure 1. We observe that the directional stabilisation by means of the structure tensor allows a piecewise constant segmentation, where the coherence-enhancing shock filter closes interrupted lines without affecting semantically important singularities in the fingerprint. A conventional shock filter, on the other hand, may even widen the gaps and disconnect previously connected structures.

In Figure 2, we compare our novel shock filter with coherence-enhancing diffusion filtering [21, 22]. While both filters have been designed for the processing of flow-like features, we observe that the diffusion filter acts smoothing while the shock filter has very pronounced sharpening properties. In certain applications

Fig. 1. Comparison between conventional and coherence-enhancing shock filtering. **(a) Left:** Fingerprint image, 186×186 pixels. **(b) Middle:** Stationary state using the shock filter (5) with $\sigma = 1.5$. **(c) Right:** Stationary state using coherence-enhancing shock filtering with $\sigma = 1.5$ and $\rho = 5$.

the latter one is thus an interesting alternative to coherence-enhancing diffusion filtering.

Figure 3 shows the influence of the structure scale σ. It is the main parameter of the filter and determines the resulting line thickness. Using values that are larger than the thickness of the initial flow lines, one obtains very interesting, almost artistic simplifications of flow-like images. The CPU time for filtering such a 512×512 colour image on a PC with AMD Athlon 1800+ processor is less than 10 seconds.

7 Summary and Conclusions

By combing the sharpening qualities of shock filters with the robust orientation estimation of the structure tensor, we have introduced a novel class of image enhancement methods: coherence-enhancing shock filters. These filters are designed for visualising flow-like structures. They inherit a number of interesting stability properties from conventional shock filters. These properties distinguish them from most Fourier- and wavelet-based enhancement methods as well as from classical methods in the spatial domain such as unsharp masking: Gibbs-like artifacts do not occur, a discrete maximum-minimum principle holds, and the total variation is not increasing. Experiments demonstrate that a piecewise constant segmentation is obtained within finite time such that there is no need to specify a stopping time. The process involves one main parameter: the structure scale σ which determines the distance between adjacent flow lines in the resulting image. Our experiments show that coherence-enhancing shock filters produce sharper results than coherence-enhancing diffusion filters, and that they outperform conventional shock filters when flow-like patterns are to be processed.

In out future work we intend to explore a number of application fields for coherence-enhancing shock filters. It can be expected that they are particularly well-suited for some computer graphics applications such as flow visualisation.

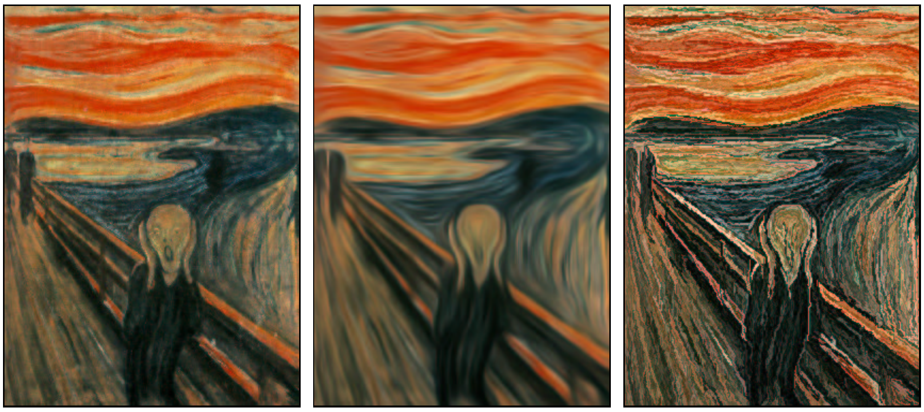

Fig. 2. Comparison between coherence-enhancing diffusion and coherence-enhancing shock filtering. **(a) Left:** Painting by Munch (The Cry, 1893; National Gallery, Oslo), 277×373 pixels. **(b) Middle:** Coherence-enhancing diffusion, $\sigma = 0.5$, $\rho = 5$, $t = 10$. **(c) Right:** Coherence-enhancing shock filtering, stationary state, $\sigma = 0.5$, $\rho = 5$. This is a colour image.

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Fig. 3. Increasing the structure scale σ creates artistic effects. **(a) Left:** Mandrill, 512×512 pixels. **(b) Middle:** Coherence-enhancing shock filtering, $\sigma = 2$, $\rho = 5$, $t = 10$. **(c) Right:** Ditto with $\sigma = 4$. This is a colour image.

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